basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12/*GRAAD* 12

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/ *PUNTE*: 150

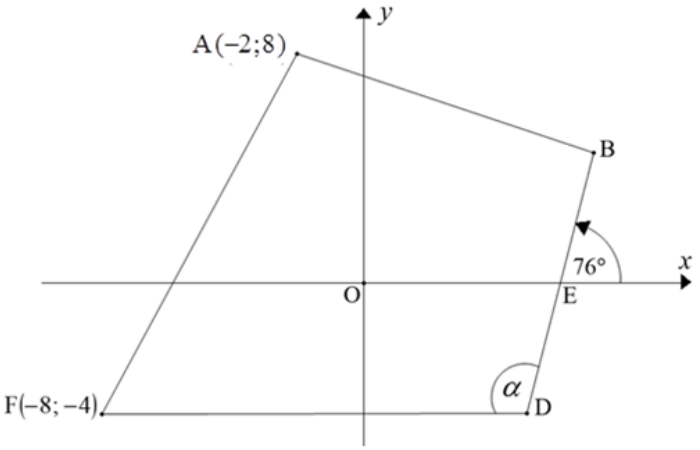
CODE/KODE	EXPLANATION/VERDUIDELIKING
A	Accuracy/*Akkuraatheid*
AO	Answer Only/*Slegs antwoord*
CA	Consistent accuracy/*Konsekwente akkuraatheid*
I	Identity/*Identiteit*
F	Correct Formula/*Korrekte formule*
M	Method/*Metode*
NPR	No penalty for rounding/*Geen penalisering vir afronding*
NPU	No penalty for units/*Geen penalisering vir eenhede*
R	Rounding/*Afronding*
RE	Reason/*Rede*
S	Simplification/*Vereenvoudiging*
SF	Substitution in correct Formula /*Vervanging in korrek Formule*
ST	Statement/*Bewering*
ST/RE	Statement with Reason /*Bewering met rede*

These marking guidelines consists of 29 pages.
Hierdie nasienriglyne bestaan uit 29 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy to be applied as indicated on the marking guidelines.
- # Questions where Tolerance Range will be applied are Q4.2.2, Q6.3, Q10.1.3 and Q11.2.3.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.*
- *Volgehoue akkuraatheid sal toegepas word soos op die nasienriglyne van aangedui.*
- *# Vrae waar Toleransie Wydte toegepas word is V*4.2.2, *V*6.3, *V*10.1.3 *en V*11.2.3.

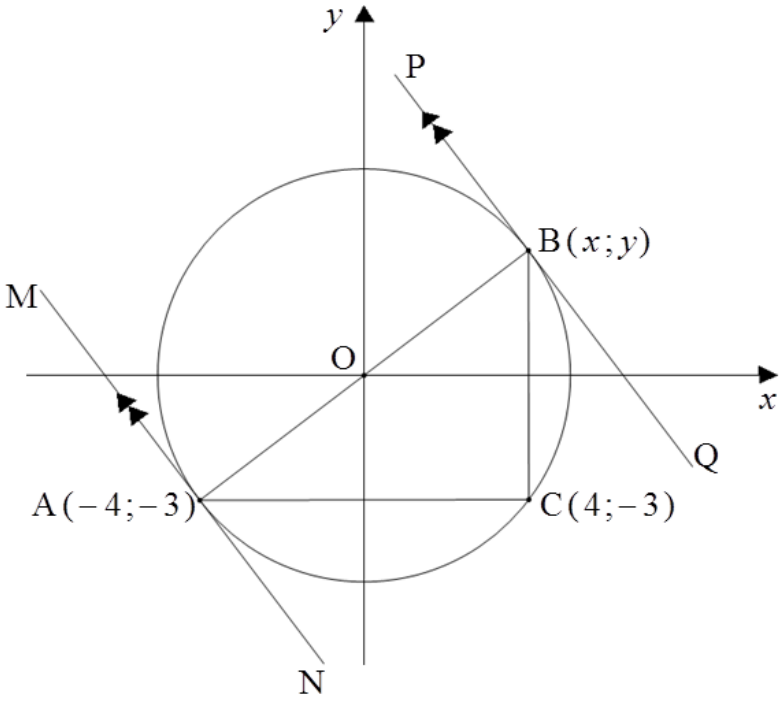
QUESTION/*VRAAG* 1



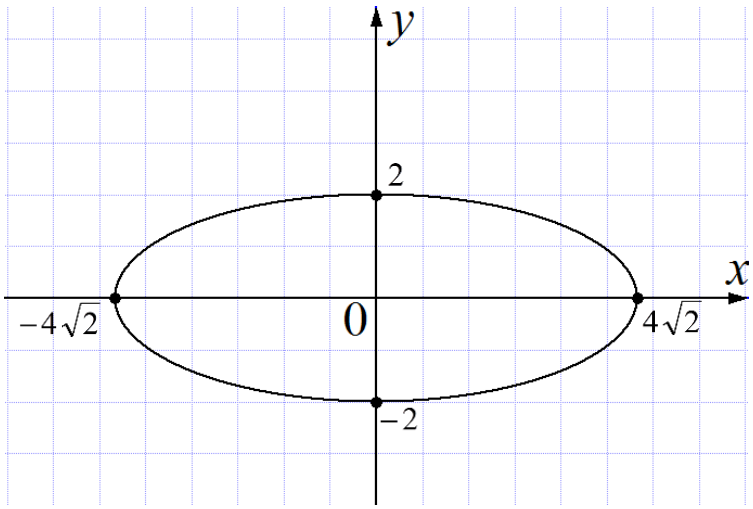
1.1	$\hat{OED} = 76°$ $\therefore\ \alpha = 104°$	✓ angle/*hoek* **A** **AO: Full marks/*Volpunte*** (1)	
1.2	$AF = \sqrt{(-2-(-8))^2 + (8-(-4))^2}$ $= \sqrt{36+144}$ $= \sqrt{180}$ $= 6\sqrt{5}$	✓ **SF** **A** ✓ length in simplified surd form/ *lengte in vereenvoudigde wortelvorm* **CA** **AO: Full marks/*Volpunte*** (2)	**L2**
1.3	$m = \tan\theta$ $= \tan 76°$ ≈ 4	✓ **SF** **A** ✓ gradient/*gradiënt* (rounded/*afgerond*) **AO: Full marks/*Volpunte*** (2)	**L2**
1.4	$M_{AF}\left(\frac{-2+(-8)}{2}; \frac{8+(-4)}{2}\right)$ $M_{AF}(-5\ ;\ 2)$	✓ **SF** **A** ✓ **S** coordinates of/*koördinate van* M_{AF} **A** **AO: Full marks/*Volpunte*** (2)	**L1**

1.5	$m_{AF} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{8-(-4)}{-2-(-8)}$ **O**R/OF $= \frac{8-2}{-2-(-5)}$ **O**R/OF $= \frac{-4-2}{-8-(-5)}$ $= 2$ $\therefore m_{perpend} = -\frac{1}{2}$ $y - 2 = -\frac{1}{2}[x-(-5)]$ **OR/*OF*** $2 = -\frac{1}{2}(-5) + c$ $y = -\frac{1}{2}x - \frac{5}{2} + 2$ **OR/*OF*** $c = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}x - \frac{1}{2}$	✓ **SF** **A** ✓ gradient of/*gradiënt van* m_{AF} C**A** ✓ gradient of/*gradiënt van* $m_{perpend}$ C**A** ✓ **SF** **CA from Q 1.4** ✓ equation/*vergelyking* C**A** (5)	
		[12]	

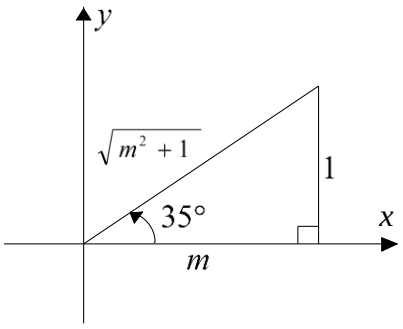
QUESTION/*VRAAG* 2



2.1.1	$x^2 + y^2 = (-4)^2 + (-3)^2$ **OR/*OF*** $x^2 + y^2 = (4)^2 + (-3)^2$ $\therefore x^2 + y^2 = 25$ **OR/*OF*** $y = \pm\sqrt{25 - x^2}$	✓ **SF** **A** ✓ equation/*vergelyking* **A** **AO: Full marks/*Volpunte*** (2)	**L1**
2.1.2(a)	B(4 ; 3)	✓ coordinates of/ *koördinate van* B **A** (1)	
2.1.2(b)	$\therefore m_{PQ} = -\frac{4}{3}$	✓ gradient of/*gradiënt van* PQ **A** (1)	
2.1.3	$y - 3 = -\frac{4}{3}(x-4)$ $y = -\frac{4}{3}x + \frac{16}{3} + 3$ $y = -\frac{4}{3}x + \frac{25}{3}$ **OR/*OF***	✓ **SF** **CA** **from/*van* Q/ *V* 2.1.2(a) & (b)** ✓ **S** **CA** ✓ equation/*vergelyking* **CA** **OR/*OF***	**L2**

	$y = mx + c$ $3 = -\frac{4}{3}(4) + c$ $c = \frac{25}{3}$ $y = -\frac{4}{3}x + \frac{25}{3}$	✓ **SF CA from/*van* Q/*V* 2.1.2(a) & (b)** ✓ value of/*waarde van c* **CA** ✓ equation/*vergelyking* **CA** (3)	
2.2.1	$x^2 + 8y^2 - 32 = 0$ $\frac{1}{32}x^2 + \frac{8}{32}y^2 = \frac{32}{32}$ $\frac{x^2}{\left(4\sqrt{2}\right)^2} + \frac{y^2}{2^2} = 1$	✓dividing both sides/*deel albei sye* by/*deur* 32 and/en transposing/*oordra* **A** ✓correct surd form/ *korrekte wortelvorm* **CA** **Accept:** $\frac{x^2}{\left(\sqrt{32}\right)^2} + \frac{y^2}{2^2} = 1$ **AO: Full marks/*Volpunte*** (2)	**L1**
2.2.1		**CA from/*van* Q/V 2.2.1** ✓both *y*-intercepts/ *beide y-afsnitte* **CA** ✓ both *x*-intercepts/ *beide x-afsnitte* **CA** ✓ elliptical shape *elliptiese vorm* **CA** (3)	
		[12]	

QUESTION/*VRAAG* 3

3.1.1	$\sin 3\alpha = \sin 3(32°)$ $\approx 0{,}99$	✓ value of /*waarde van* $\sin 3\alpha$ **A** **NPR** (1)	**L1**
3.1.2	$\frac{\sec^2\theta - 1}{\tan\alpha} = \frac{\sec^2 20° - 1}{\tan 32°}$ $= \frac{\left(\frac{1}{\cos 20°}\right)^2 - 1}{\tan 32°}$ $\approx 0,21$ **OR/*OF*** $\frac{\sec^2\theta - 1}{\tan\alpha} = \frac{\tan^2 20°}{\tan 32°}$ $\approx 0,21$	✓ I $\left(\frac{1}{\cos 20°}\right)$ **A** ✓ value of/*waarde van* $\frac{\sec^2\theta - 1}{\tan\alpha}$ **CA** **OR/*OF*** ✓ **I** **A** ✓ value of/*waarde van* $\frac{\sec^2\theta - 1}{\tan\alpha}$ **CA** **NPR** **AO: Full marks/*Volpunte*** (2)	**L1**
3.2.1	 $r = \sqrt{m^2+1}$ $\sin 35° = \frac{1}{\sqrt{m^2+1}}$	✓ r **A** ✓ definition of/ *definisie van* sin **CA** **AO: Full marks/*Volpunte*** (2)	**L2**

3.2.2	$\left(\cos\frac{29}{36}\pi\right)\left(\tan\frac{7}{36}\pi\right)$ $=(\cos 145°)(\tan 35°)$ $=(-\cos 35°)\left(\frac{\sin 35°}{\cos 35°}\right)$ $=-\sin 35°$ $=-\frac{1}{\sqrt{m^2+1}}$ **OR/*OF*** $\left(\cos\frac{29}{36}\pi\right)\left(\tan\frac{7}{36}\pi\right)$ $=(\cos 145°)(\tan 35°)$ $=(-\cos 35°)(\tan 35°)$ $=\left(-\frac{m}{\sqrt{m^2+1}}\right)\left(\frac{1}{m}\right)$ $=-\frac{1}{\sqrt{m^2+1}}$ **OR/*OF*** $\left(\cos\frac{29}{36}\pi\right)\left(\tan\frac{7}{36}\pi\right)$ $=\left[\cos\left(\pi-\frac{29}{36}\pi\right)\right]\left(\tan\frac{7}{36}\pi\right)$ $=\left(-\cos\frac{7}{36}\pi\right)\left(\frac{\sin\frac{7}{36}\pi}{\cos\frac{7}{36}\pi}\right)$ $=-\sin\frac{7}{36}\pi=-\sin 35°$ $=-\frac{1}{\sqrt{m^2+1}}$	✓ changing to degrees/ *verander na grade* **A** ✓ $-\cos 35°$ **CA** ✓ **I** **A** ✓ **S** **CA** ✓ ratio/*verhouding* **CA** **OR/*OF*** ✓ changing to degrees/ *verander na grade* **A** ✓ $-\cos 35°$ **CA** ✓ ratio of/*verhouding van* cos **CA** ✓ ratio of/*verhouding van* tan **A** ✓ **S** **CA** **OR/*OF*** ✓ reduction/*reduksie* **A** ✓ $-\cos\frac{7}{36}\pi$ **CA** ✓ **I** **A** ✓ **S** **CA** ✓ ratio of/*verhouding van* sine **CA** (5)	

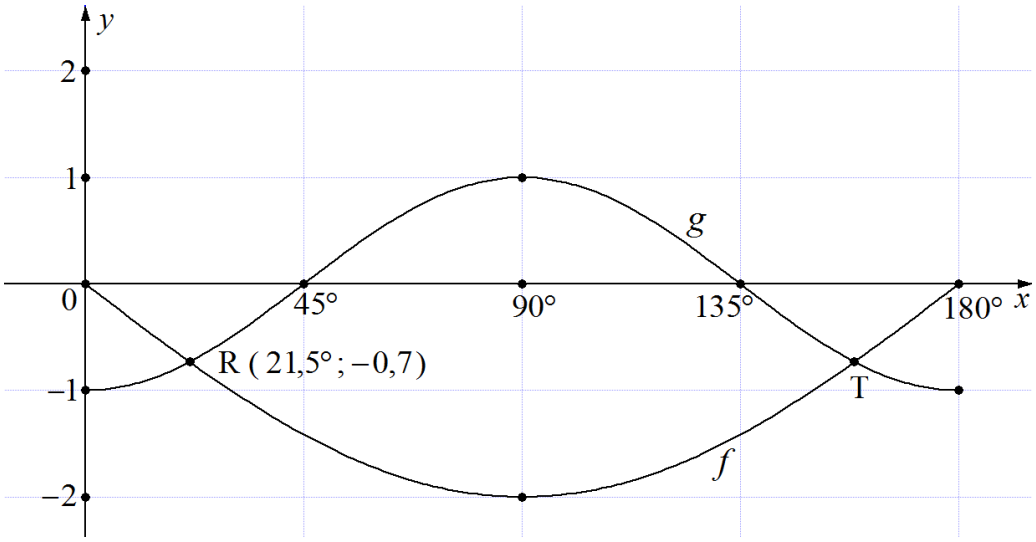
3.3.1	$2\cos\theta + \sin\theta = 0$ $\sin\theta = -2\cos\theta$ $\frac{\sin\theta}{\cos\theta} = \frac{-2\cos\theta}{\cos\theta}$; $\cos\theta \neq 0$ $\therefore \tan\theta = -2$ **OR/*OF*** $2\cos\theta = -\sin\theta$ $\frac{2\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta}$; $\sin\theta \neq 0$ $\cot\theta = -\frac{1}{2}$ $\frac{1}{\tan\theta} = -\frac{1}{2}$ $\tan\theta = -2$	✓ transposing/*oordra* **A** ✓ **M** dividing both sides by/*deel beide kante deur* $\cos\theta$ **A** (2) **OR/*OF*** ✓ transposing/*oordra* **A** ✓ **M** dividing both sides by/*deel beide kante deur* $\sin\theta$ **A** (2)	
3.3.2	$\tan\theta = -2$ Ref/verw. $\angle \approx 63,43°$ $\theta \approx 180° - 63,43°$ or/*of* $\theta \approx 360° - 63,43°$ $\therefore \theta \approx 116,57°$ or/*of* $\theta \approx 296,57°$	✓ ref/*verw* **A** ✓ 2nd & 4th /*2de en 4de* quadrants/*kwadrante* **A** ✓ $116,57°$ **CA** ✓ $296,57°$ **CA** **AO: Full marks/*Volpunte*** (4)	
		[16]	

QUESTION/*VRAAG* 4

4.1.1	$\cot^2 2\beta - \text{cosec}^2 2\beta = -1$	✓ -1 **A** (1)	
4.1.2	$\tan^2 A \cdot \text{cosec}^2 A - \cos 2\pi$ $= \frac{\sin^2 A}{\cos^2 A} \cdot \frac{1}{\sin^2 A} - 1$ $= \frac{1}{\cos^2 A} - 1$ $= \frac{1 - \cos^2 A}{\cos^2 A} = \frac{\sin^2 A}{\cos^2 A}$ $= \tan^2 A$ **OR/*OF*** $\tan^2 A \cdot \text{cosec}^2 A - \cos 2\pi$ $= \frac{\sin^2 A}{\cos^2 A} \cdot \frac{1}{\sin^2 A} - 1$ $= \frac{1}{\cos^2 A} - 1$ $= \sec^2 A - 1$ $= \tan^2 A$ **OR/*OF*** $\tan^2 A \cdot \text{cosec}^2 A - \cos 2\pi$ $= \tan^2 A(1 + \cot^2 A) - 1$ $= \tan^2 A + \tan^2 A \cdot \cot^2 A - 1$ $= \tan^2 A + 1 - 1$ $= \tan^2 A$	✓ **I** $\left(\frac{\sin A}{\cos A}\right)$ **A** ✓ **I** **A** ✓ 1 **A** ✓ **I** **CA** ✓ **S** **CA** **OR/*OF*** ✓ **I** $\left(\frac{\sin A}{\cos A}\right)$ **A** ✓ **I** **A** ✓ 1 **A** ✓ **I** **CA** ✓ **S** **CA** **OR/*OF*** ✓ **I** **A** ✓ 1 **A** ✓ product/*produk* **CA** ✓ 1 **CA** ✓ **S** **CA** (5)	

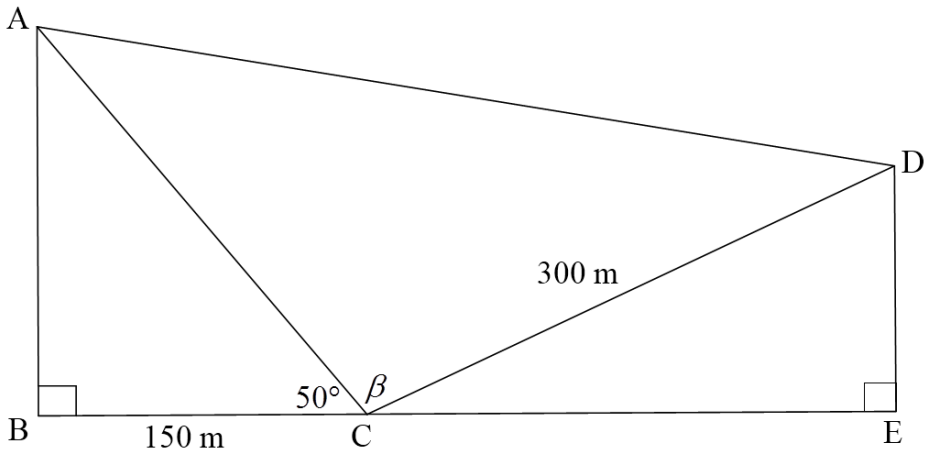
4.2.1	$\sec 60° = 2$	✓ 2 **A** (1)	
4.2.2 #	$\operatorname{cosec}(180° + \theta)\cdot\sin(360° - \theta) - [\sin(180° + \theta)]^{\sec 60°} = \cos^2\theta$ L.H.S/*LK* $= \operatorname{cosec}(180° + \theta)\cdot\sin(360° - \theta) - [\sin(180° + \theta)]^2$ $= (-\operatorname{cosec}\theta)\cdot(-\sin\theta) - \sin^2(180° + \theta)$ $= \left(-\frac{1}{\sin\theta}\right)\cdot(-\sin\theta) - (-\sin\theta)^2$ $= 1 - \sin^2\theta = \cos^2\theta =$ R.H.S./*RK*	✓ $-\operatorname{cosec}\theta$ **A** ✓ $-\sin\theta$ **A** ✓ **I** **A** ✓ $-\sin\theta$ **A** ✓ **S** $1 - \sin^2\theta$ **CA** (5)	
		[12]	

QUESTION/*VRAAG* 5



5.1	360°	✓ period/*periode* **A** (1)	
5.2	$a = -2$ $b = 2$	✓ value of/*waarde van a* **A** ✓ value of/*waarde van b* **A** (2)	
5.3	$T(158,5° ; -0,7)$	✓ *x* coordinate/*koördinaat* **A** ✓ *y* coordinate/*koördinaat* **A** **Accept:** $T(159,5° ; -0,7)$ (2)	
5.4.1	$135° < x < 180°$ **OR/*OF*** $x \in (135°;180°)$	✓ end points/*eindpunte* **A** ✓ correct notation/*korrekte notasie* **A** (2)	
5.4.2	$x = 45°$ or/*of* $x = 135°$	✓ $x = 45°$ **A** ✓ $x = 135°$ **A** (2)	
		[9]	

QUESTION/*VRAAG* 6

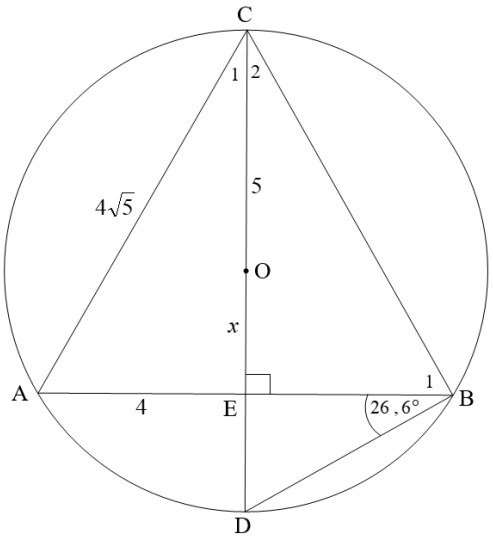


6.1	$\cos 50° = \frac{150}{AC}$ $AC = \frac{150}{\cos 50°}$ $\approx 233,36\,m$ **OR/*OF*** $AB = 150\tan 50°$ $AC = \sqrt{(150\tan 50°)^2 + (150)^2}$ $\approx 233,36\,m$ **OR/*OF*** $B\hat{A}C = 40°$ $\sin 40° = \frac{150}{AC}$ $AC = \frac{150}{\sin 40°}$ **OR/*OF*** = 150 cosec40° $\approx 233,36\,m$ **OR/*OF*** $B\hat{A}C = 40°$ $\frac{150}{\sin 40°} = \frac{AC}{\sin 90°}$ $AC = \frac{150\sin 90°}{\sin 40°}$ $\approx 233,36\,m$	✓ trig ratio/*verhouding* **A** ✓ the subject/ *die onderwerp* AC **CA** ✓ length of/*lengte van* AC **CA** **OR/*OF*** ✓ Length of AB **A** ✓ **M** **CA** ✓ length of/*lengte van* AC **CA** **OR/*OF*** ✓ trig ratio/*verhouding* **A** ✓ the subject/ *die onderwerp* AC **CA** ✓ length of/*lengte van* AC **CA** **OR/*OF*** ✓ **SF** **A** ✓ the subject/ *die onderwerp* AC **CA** ✓ length of/*lengte van* AC **CA** **NPR** (3)	

6.2	Area of/*oppervlakte van* $\Delta ACD = \frac{1}{2}\times AC\times CD\sin\beta$ $3,3648\times10^4 = \frac{1}{2}\times 233,36\times 300\times\sin\beta$ $\sin\beta = \frac{3,3648\times10^4}{35004} = \frac{2804}{2917}\approx 0,96126157$ ref / *verw.*$\angle\approx 74°$ $\beta\approx 180° - 74°$ $\therefore \quad = 106°$ **OR/*OF*** Area of/*oppervlakte van* $\Delta ACD = \frac{1}{2}\times AC\times CD\sin\beta$ $3,3648\times10^4 = \frac{1}{2}\times 233,36\times 300\times\sin\beta$ $3,3648\times10^4 = 35\,004\sin\beta$ $\sin\beta = \frac{3,3648\times10^4}{35004}$ ref / *verw.*$\angle\approx 74°$ $\beta\approx 180° - 74°$ $\therefore \quad = 106°$	✓ **F** **A** ✓ **SF** **CA** **from/*van* Q/*V* 6.1** ✓**S** **CA** ✓ ref./ *verw.*$\angle$ **CA** ✓ size of/*grootte van* β **CA** **OR/*OF*** ✓ **F** **A** ✓ **SF** **CA** **from/*van* Q/*V* 6.1** ✓**S** **CA** ✓ ref./ *verw.*$\angle$ **CA** ✓ size of/*grootte van* β **CA** **NPR** (5)	
6.3 #	$AD^2 = AC^2 + DC^2 - 2AC\cdot DC\cos A\hat{C}D$ $= (233,36)^2 + (300)^2 - 2(233,36)(300)\cos106°$ $\approx 183050,5296$ $\therefore AD\approx 427,84\,m$	✓ **F** **A** ✓ **SF** **CA** **from/*van* Q/*V* 6.1, 6.2** ✓ **S** **CA** ✓ length of /*lengte van* AD **CA** **NPR** (4)	
		[12]	

QUESTION/*VRAAG* 7

7.1	Perpendicular to the chord/*loodreg op die koord*	✓ **ST** **A** (1)	



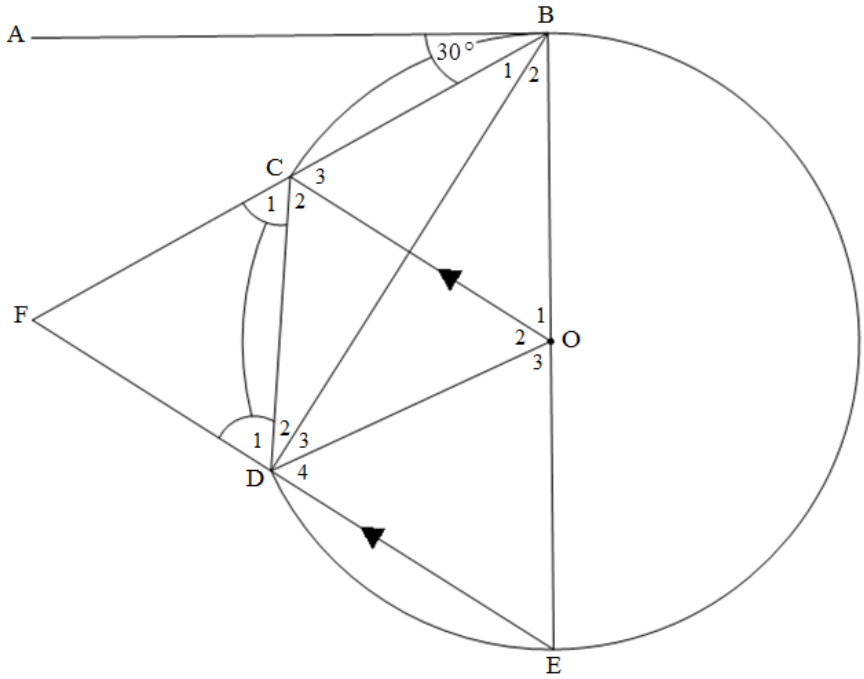
7.2.1(a)	$\hat{C}_1 = 26,6^o$ [$\angle^s$ in the same segment (arc or chord)/ $\angle^e$*in dieselfde segment* (*boog of koord*)] **OR/*OF*** $\sin \hat{C}_1 = \frac{4}{4\sqrt{5}}$ $\therefore \hat{C}_1 = 26,6^o$	✓ **ST** **A** ✓ **RE** **A** **OR/*OF*** ✓ ratio/*verhouding* **A** ✓ size of/ *grootte van* $\hat{C}_1$ **A** (2)	
7.2.1(b)	$\hat{A} = 180° - 90° - 26,6°$ [sum/*som* $\angle^s$ Δ] $= 63,4^o$ **OR/*OF*** $90° = \hat{A} + 26,6°$ [ext. $\angle$ of Δ = sum of 2 int. opp. $\angle$s / *buite* $\angle$ *van* Δ = *som teenrst binne* $\angle e$] $\hat{A} = 63,4^o$ **OR/*OF*** $\cos A = \frac{4}{4\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\therefore A = \cos^{-1}\left(\frac{\sqrt{5}}{5}\right) = 63,4°$	✓ **ST** **CA from/*van* Q/V 7.2.1(a)** **OR/*OF*** ✓ **ST** **CA from/*van* Q/V 7.2.1(a)** **OR/*OF*** ✓ **ST** **A** (1)	

7.2.1(c)	$\hat{B}_1 = 90^o - 26,6°$ [∠in semi circle/*in halfsirkel*] $= 63,4^o$ **OR/*OF*** $\cos \hat{B}_1 = \frac{4}{4\sqrt{5}}$ $\therefore \hat{B}_1 = 63,4°$ **OR/*OF*** $\hat{A} = \hat{B}_1 = 63,4°$ [∠s opp. equal sides/*∠e teenoor gelyke sye* / OE ⊥ AB and/ *en* AE = EB]	✓ **ST** **A** ✓ **RE** **A** **OR/*OF*** ✓ cos ratio/ *verhouding* **A** ✓ **ST** **A** **OR/*OF*** ✓ **ST** **A** ✓ **RE** **A** (2)	
7.2.2(a)	AE = 4 units/*eenhede*	✓ length of /*lengte van* AE **A** (1)	
7.2.2(b)	ED = 5 – *x* units/*eenhede*	✓ length of /*lengte van* ED **A** (1)	
7.2.3	$4^2 + (x+5)^2 = (4\sqrt{5})^2$ $16 + x^2 + 10x + 25 = 80$ $x^2 + 10x - 39 = 0$ $(x-3)(x+13) = 0$ $x = 3$ or/*of* $x \neq -13$ **One mark penalty if a candidate include $x = -13$ as the answer/ *Een punt penalisering indien die kandidaat $x = -13$ as antwoord insluit*** **OR/*OF*** $\frac{5-x}{4} = \tan 26,6°$ $5 - x = 4\tan 26,6°$ $5 - x \approx 2,003 \approx 2$ $x \approx 2,997 \approx 3$ **OR/*OF*** $x^2 = (5)^2 - (4)^2$ $x^2 = 9$ $x = 3$ **OR/*OF***	✓ **M** **A** ✓ **S** **CA** ✓ factors/ formula *faktore/ formule* **CA** ✓ correct value of / *korrekte waarde van x* **CA** **OR/*OF*** ✓ tan ratio/verhouding **A** ✓ **S** **CA** ✓ **S** **CA** ✓ correct value of / *korrekte waarde van x* **CA** **OR/*OF*** ✓ Th. of Pythagoras **A** ✓ subst./*verv.* **CA** ✓ S **CA** ✓ correct value of / *korrekte waarde van x* **CA** **OR/*OF***	

	$4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $h^2 - 10h + 16 = 0$ $(h-8)(h-2) = 0$ $h = 8$ or $h = 2$ EC $= x + 5$ $8 = x + 5$ $\therefore x = 3$	✓ **SF** **A** ✓ factors/ formula *faktore/ formule* **CA** ✓ both values of/ *beide waardes van h* **CA** ✓ correct value of / *korrekte waarde van x* **CA**	
	OR/*OF*	**OR/*OF***	
	$\sin 63,4° = \frac{x+5}{4\sqrt{5}}$ $8 \approx x + 5$ $\therefore x \approx 3$	✓ sin ratio/ *verh* **A** ✓ substitution/ *vervanging* **A** ✓ S **CA** ✓ correct value of / *korrekte waarde van x* **CA**	
	OR/*OF*	**OR/*OF***	
	$CE^2 = \left(4\sqrt{5}\right)^2 - (4)^2$ $CE^2 = 64$ $CE = 8$ $x = 8 - 5$ $\therefore x = 3$	✓ Theorem of Pyth. **A** ✓ substitution/ *vervanging* **A** ✓ value of/ waarde van CE **CA** ✓ correct value of / *korrekte waarde van x* **CA** **AO: Full marks/*Volpunte*** (4)	
		[12]	

QUESTION/*VRAAG* 8

8.1	90° / Right angle/ *Regte hoek*	✓ 90° / Right angle/ *Regte hoek* **A** (1)	

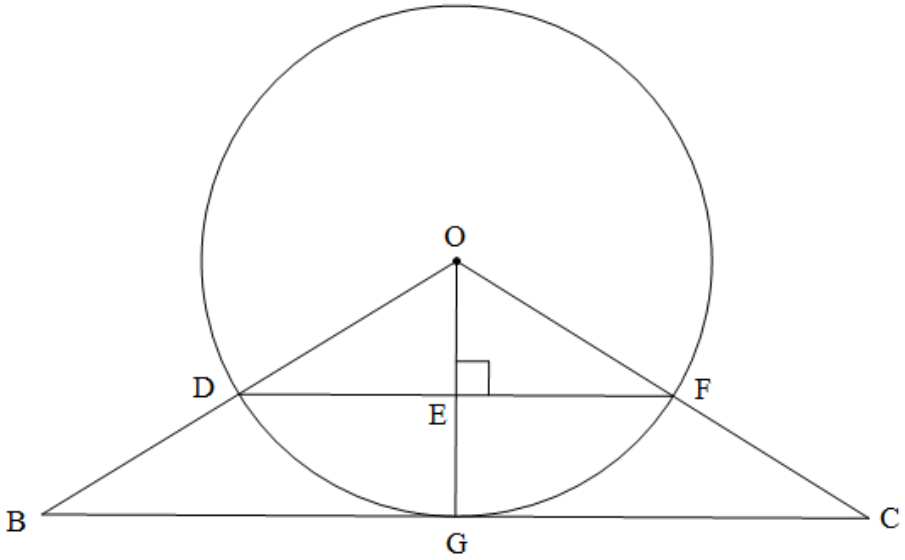


8.2.1(a)	$\hat{CBO} = 90^o - 30^o$ [tan ⊥ radius (diameter)/ *raaklyn ⊥ radius (middellyn)*] $= 60^o$	✓ **ST** **A** ✓ **RE** **A** (2)	
8.2.1(b)	$\hat{D}_2 = 30^o$ [tan – chord/*raaklyn – koord*]	✓ **ST** **A** ✓ **RE** **A** (2)	
8.2.1(c)	$\hat{O}_1 = 60^o$ [∠at centre = 2∠at circumfere nce/ *middelpunt∠ = 2 omtreks∠*] **OR/ *OF*** $O\hat{B}A = 90°$ [rad ⊥ tan/*raaklyn*] $\therefore O\hat{B}C = 60°$ $\therefore O\hat{B}C = \hat{C}_3 = 60°$ [∠s opp. equal sides, OC and OB radii / *∠e teenoor gelyke sye, OC en OB radii*] $\hat{O}_1 = 180° - 120° = 60°$ [sum of ∠s of Δ / *som van ∠e van Δ*]	✓ **ST** **CA from/*van* Q/*V* 8.2.1 *(a)*** ✓ **RE** **A** **OR/ *OF*** ✓ **ST** **CA from/*van* Q/*V* 8.2.1 *(a)*** ✓ **ST** **A** (2)	

8.2.1(d)	$\hat{O}_1 = \hat{E} = 60^o$ $\begin{bmatrix}\text{corresp } \angle s / \textit{ooreenk } \angle e \\ CO \parallel FE\end{bmatrix}$ $\hat{D}_4 = \hat{E} = 60^o$ $\angle s \text{ opp.} = \text{sides} / \textit{\angle e teenoor = sye}]$ $\hat{O}_2 = \hat{D}_4 = 60^o$ [Alt $\angle s$ /*verw.∠e*, $CO \parallel FE$]	✓ **ST** **A** ✓ **ST** **A** ✓ **ST** **A** (3)	
8.2.2	$\hat{C}_1 = \hat{E} = 60^o$ $\begin{bmatrix}\text{ext } \angle \text{ of cyclic quad} / \\ \textit{buite } \angle \textit{van koorde vierh}\end{bmatrix}$. $\hat{D}_1 = C\hat{B}O = 60^o$ $\begin{bmatrix}\text{ext } \angle \text{ of cyclic quad} / \\ \textit{buite } \angle \textit{van koorde vierh.}\end{bmatrix}$ $\therefore FC = FD$ $\begin{bmatrix}\text{sides opposite} = \angle s / \\ \textit{sye teenoor} = \angle e\end{bmatrix}$ **OR/*OF*** $FB = FE$ $[\text{sides opp.} = \angle s / \textit{sye teenoor} = \angle e]$ $DE = CB$ $[\text{equilateral } / \textit{gelyksydige } \Delta]$ $FB - CB = FE - DE$ $\therefore FC = FD$	✓**ST** **CA** ✓**RE** **A** ✓**ST** **CA** ✓ **RE** **A** **OR/*OF*** ✓ **ST** **CA** ✓**RE** **A** ✓ **ST** **CA** ✓ **M** **A** (4)	
		[14]	

QUESTION/*VRAAG* 9

9.1	Parallel to the third side/*Ewewydig aan die derde sy*	✓ **ST** **A** (1)	

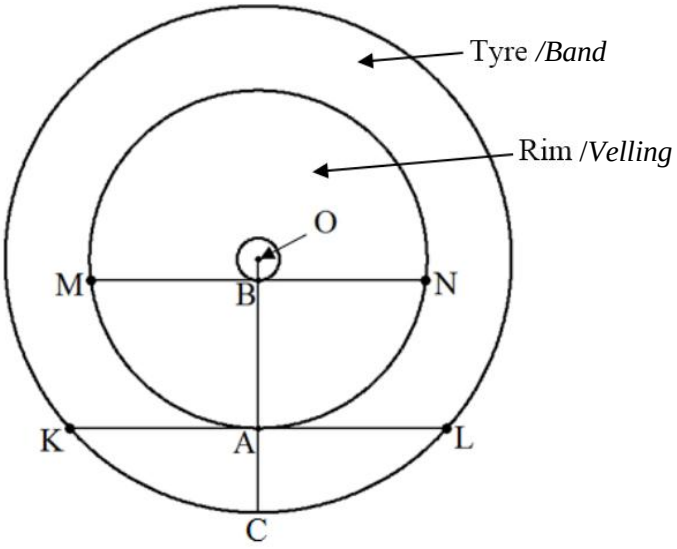


9.2.1	$\hat{OGC} = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $\hat{OEF} = 90°$ [OG ⊥ GC] $\therefore$ DF || BC [corr. ∠s are equal/*ooreenk. ∠e is gelyk*]	✓ **ST** **A** ✓**RE** **A** ✓ **RE** **A** (3)	
9.2.2 (a)	BC : DF = 10 : 6 = 5 : 3 **OR/*OF*** $\Delta ODF /// \Delta OBC$ $\frac{OD}{OB} = \frac{OF}{OC} = \frac{DF}{BC} = \frac{3}{5}$ $\therefore$ BC : DF = 5 : 3 **OR/*OF*** BC : DF = 10 : 6	✓ proportionality/*eweredigh* **A** ✓ simplified ratio/ *vereenv.verhouding* **A** **OR/*OF*** ✓ proportionality/*eweredigh* **A** ✓ simplified ratio/ *vereenv.verhouding* **A** **AO: Full marks/*Volpunte*** (2)	

9.2.2 (b)	$\frac{EG}{OG}=\frac{2}{5}$ $\frac{EG}{6}=\frac{2}{5}$ $EG=\frac{12}{5}$ units / *eenhede* **OR / *OF*** $2,4$ units /*eenhede* **OR/*OF*** $\frac{OD}{OB}=\frac{OE}{OG}$ $\frac{3}{5}=\frac{OE}{6}$ $OE=\frac{18}{5}=3,6$ $EG=6-3,6=2,4$	✓proportionality/*eweredigh* **A** ✓substitution/*vervanging* **A** ✓length of /*lengte van* EG **CA** **OR/*OF*** ✓ proportionality/*eweredigh* **A** ✓ substitution/*vervanging* **A** ✓ length of /*lengte van* EG **CA** (3)	
9.2.2 (c)	$OE=6-2,4=3,6$ units/*eenhede* $\frac{\text{Area (}Opp.\text{) }\Delta OBG}{\text{Area (}Opp.\text{) }\Delta ODE}=\frac{\frac{1}{2}(10)(6)\sin B\hat{O}G}{\frac{1}{2}(6)(3,6)\sin B\hat{O}G}$ $=\frac{25}{9}$ **OR/*OF*** $\frac{\text{Area (}Opp.\text{) }\Delta OBG}{\text{Area (}Opp.\text{) }\Delta ODE}=\frac{\frac{1}{2}(BG)(OG)}{\frac{1}{2}(DE)(OE)}$ $=\frac{5}{3}\times\frac{6}{3,6}$ $=\frac{25}{9}$ **OR/*OF*** $\frac{\text{Area (}Opp.\text{) }\Delta OBG}{\text{Area (}Opp.\text{) }\Delta ODE}=\frac{\frac{1}{2}(4,8)(3,6)}{\frac{1}{2}(8)(6)}$ $=\frac{24}{8,64}$ $\approx 2,78$	✓ length of /*lengte van OE* **CA from /*van* Q/*V* 9.2.2 (b)** ✓ **SF** **CA** ✓ value/*waarde* **CA** **OR/*OF*** ✓ length of /*lengte van* OE **CA from/*van* Q/*V* 9.2.2 (b)** ✓ **SF** **CA** ✓ value/*waarde* **CA** **OR/*OF*** ✓ length of /*lengte van* DE **CA from/*van* Q/*V* 9.2.2 (b)** ✓ **SF** **CA** ✓ value/*waarde* **CA** (3)	

9.3	In ΔDOE and/*en* ΔBOG $\hat{D}OE = \hat{B}OG$ [common /*gemene∠*] $O\hat{E}D = O\hat{G}B = 90°$ [proved / *bewys*] $\therefore O\hat{D}E = O\hat{B}G$ [3rd/3*de* ∠] $\therefore \Delta DOE \,|||\, \Delta BOG$ ∠∠∠ **OR/*OF*** In ΔDOE and/*en* ΔBOG $\frac{OE}{OG} = \frac{3,6}{6} = \frac{3}{5}$ $\frac{OD}{OB} = \frac{6}{10} = \frac{3}{5}$ $\frac{DE}{BG} = \frac{3}{5}$ $\therefore \Delta DOE \,|||\, \Delta BOG$ [corr.sides are in proportion/ *ooreen. sye is eweredig*]	✓ **ST** **A** ✓ **ST** **A** ✓ **RE** **A** **OR/*OF*** ✓ one ratio/*verh* **A** ✓ two ratios/*verh* **A** ✓ **RE** **A** (3)	
		[15]	

QUESTION/*VRAAG* 10

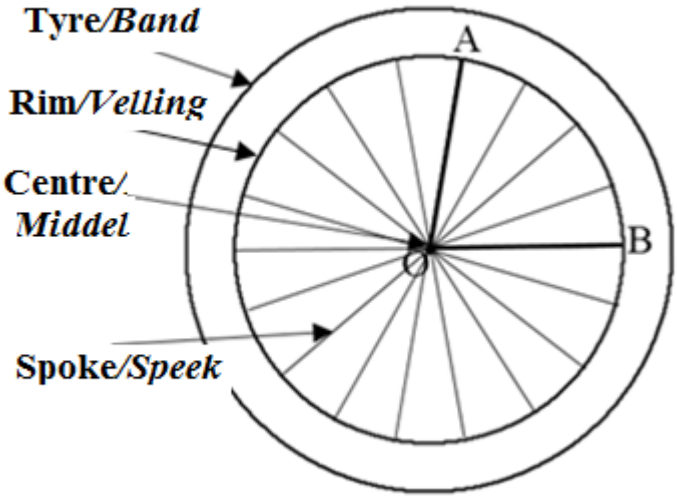


10.1.1	BC = 20 cm – 1,5 cm = 18,5 cm	✓ length of /*lengte van BC* **A** (1)	
10.1.2	$4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(40)h + (32)^2 = 0$ $4h^2 - 160h + 1024 = 0$ $4(h^2 - 40h + 256) = 0$ $4(h - 32)(h - 8) = 0$ $h = 32$ or / *of* $h = 8$ $\therefore h =$ AC = 8 cm $\therefore$ AB = 18,5 cm – 8 cm = 10,5 cm **OR/*OF*** $OC = \frac{1}{2}(40) = 20$ cm OC = OK = 20 cm radii $OK^2 - KA^2 = OA^2$ $(20)^2 - (16)^2 = OA^2$ OA = 12 cm AB = (12 – 1,5) cm = 10,5 cm **OR/OF**	✓ **F** **A** ✓ **SF** **A** ✓ factors/formula/*faktore/formule* **CA** ✓ length of /*lengte van* AC **CA** ✓ length of /*lengte van* AB **CA** **OR/*OF*** ✓ length of /*lengte van OC* **A** ✓ **SF** **A** ✓ length of /*lengte van* OA **CA** ✓ AB = 12 – 1,5 **CA** ✓ length of /*lengte van* AB **CA** **OR/OF**	

	In Δ OAL: $OC = OL = 20\,cm$ radii $OL^2 = OA^2 + AL^2$ $(20)^2 = OA^2 + (16)^2$ $400 - 256 = OA^2$ $OA = \sqrt{144} = 12$ $\therefore AB = OA - OB$ $= 12 - 1,5$ $= 10,5\,cm$	✓ length of/ *lengte van* OL **A** ✓ substitution/ *vervanging* **A** ✓ value of / *waarde van* OA **CA** ✓ **M** **CA** ✓ length of /*lengte van* AB **CA** (5)	
10.1.3	$w = 64\pi$ rad/min $= 64\pi \times 2\pi$ rad/min $= 128\pi^2$ rad/min **OR**/***OF*** $\approx 1263,13$ rad/min $\omega = 2\pi n$ $128\pi^2 = 2\pi n$ **OR**/***OF*** $1263,13 = 2\pi n$ $\frac{128\pi^2}{2\pi} = \frac{2\pi n}{2\pi}$ $\therefore n = 64\pi$ rev/min **OR**/***OF*** $\approx 201,03$ rev/min	✓ conversion **A** ✓ **SF** **CA** ✓ value of /*waarde van n* **CA** **(**3)	
10.1.4	$v = \pi D n$ $= \pi(40\,cm) \times (64\pi)$ $\therefore v = 25266,19$ cm/min **OR**/***OF*** $\therefore v = 421,10$ cm/sec **OR**/***OF*** $\therefore v = 4,21$ m/sec **OR**/***OF*** $\therefore v = 252,66$ m/min	✓ **F** **A** ✓ **SF** **CA** ✓ value of *v* **CA** ✓ unit **CA**	

	OR/*OF* $v = \pi D n$ $= \pi(40\,\text{cm}) \times (64\pi)$ $\therefore v = 25266{,}19$ cm/min **OR/*OF*** $\therefore v = 421{,}10$ cm/sec **OR/*OF*** $\therefore v = 4{,}21$ m/sec **OR/*OF*** $\therefore v = 252{,}66$ m/min	**OR/*OF*** ✓ **F** **A** ✓ **SF** **A** ✓ value of / *waarde van v* **CA** ✓ unit/ *eenheid* **CA** (4)	

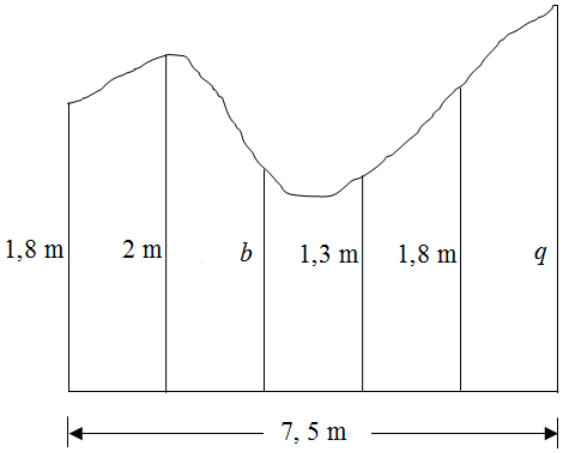
10.2



10.2.1	$A\hat{O}B = 4 \times 20° = 80° = \frac{4}{9}\pi$ $= 1,4$ **OR/*OF*** $A\hat{O}B = \frac{4}{18} \times 360° = 80° = \frac{4}{9}\pi$ $= 1,4$ **OR/*OF*** $P = 2\pi r = 2\pi(5,2) \approx 32,67.....cm$ Dist. between each spoke/ *Afst. tussen elke speek* $= \frac{32,67}{18} = 1,815...cm$ $AB = 4 \times 1,815.....$ $= 7,26....cm$ $s = r\theta$ $7,26.... = (5,2)\theta$ $\theta \approx 1,4$	✓ **M** **A** ✓ magnitude of/*grootte van* $A\hat{O}B$ **A** ✓ ∠ in radians/ *in radiale* **CA** **OR/*OF*** ✓ **M** **A** ✓ magnitude of/*grootte van* $A\hat{O}B$ **A** ✓ ∠ in radians/ *in radiale* **CA** **OR/*OF*** ✓ Perimeter/ *omtrek* **A** ✓ **SF** **A** ✓ ∠ in radians/ *in radiale* **CA** (3)	

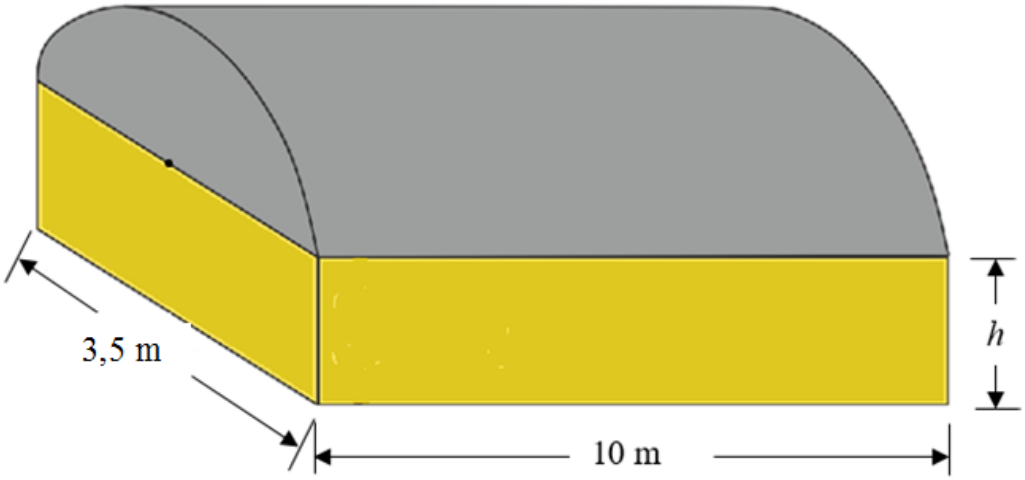
10.2.2	$s=r\theta$ $=5,2cm\times1,4$ or/*of* $5,2cm\times\frac{4}{9}\pi$ $\approx7,3cm$ **OR/*OF*** $s=\frac{4}{18}\times2\pi r$ $=\frac{4}{18}\times2\pi(5,2)$ cm $\approx7,3cm$	✓ **F** **A** ✓ **SF** **CA** ✓ arc length/*booglengte* **CA** **OR/*OF*** ✓ **M** **A** ✓ $\frac{4}{18}$ **A** ✓ arc length/ *booglengte* **CA** (3)	
10.2.3	Area of a sector/*opp. van sektor* $=\frac{rs}{2}$ $=\frac{5,2\times7,3}{2}$ $=19\text{ cm}^2$ **OR/*OF*** Area of a sector/*opp. van sektor* $=\frac{r^2s}{2}$ $=\frac{(5,2)^2\times1,4}{2}$ $=19\text{ cm}^2$ **OR/*OF*** Area of a sector/*Opp.van sektor* $=\frac{4}{18}\pi r^2$ $=\frac{4}{18}\pi(5,2)^2\text{ cm}^2$ $\approx19\text{ cm}^2$	✓ **F** **A** ✓ **SF** **CA** ✓ area of sector/*Opp.van sektor* **CA** **OR/*OF*** ✓ **F** **A** ✓ **SF** **CA** ✓ area of sector/*Opp.van sektor* **CA** **OR/*OF*** ✓ ratio/*verh* **A** ✓ substitution/ *vervang* **CA** ✓ area of sector/*opp van sektor* **CA** (3)	
		[22]	

QUESTION/*VRAAG* 11



11.1.1	$b = \sqrt{2}$ $\approx 1{,}4\,\text{m}$	✓ value of /value of *b* **A** **NPR** (1)	
11.1.2	$A_{Irr} = \frac{2}{3} \times 19{,}125\,\text{m}^2 = 12{,}75\ \text{m}^2$ $A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ $12{,}75 = 1{,}5\left(\frac{1{,}8+2}{2} + \frac{2+1{,}4}{2} + \frac{1{,}4+1{,}3}{2} + \frac{1{,}3+1{,}8}{2} + \frac{1{,}8+q}{2}\right)$ $12{,}75 = 1{,}5\left(1{,}9 + 1{,}7 + 1{,}35 + 1{,}55 + \frac{1{,}8+q}{2}\right)$ $8{,}5 = 6{,}5 + \frac{1{,}8+q}{2}$ $4 = 1{,}8 + q$ $\therefore q = 2{,}2\,\text{m}$ **OR/*OF*** $A_{Irr} = \frac{2}{3} \times 19{,}125\,\text{m}^2 = 12{,}75\ \text{m}^2$ $12{,}75 = 1{,}5\left(\frac{1{,}8+q}{2} + 2 + 1{,}4 + 1{,}3 + 1{,}8\right)$ $8{,}5 = 6{,}5 + \frac{1{,}8+q}{2}$ $4 = 1{,}8 + q$ $\therefore q = 2{,}2\,\text{m}$	✓ value of /*waarde van* A_{Irr} **A** ✓ **F** **A** ✓ value of /*waarde van* *a* **A** ✓ **SF** **CA** ✓ value of /*waarde van* *q* **CA** **OR/*OF*** ✓ value of/*waarde van* A_{Irr} **A** ✓ **F** **A** ✓ value of /*waarde van* *a* **A** ✓ **SF** **CA** ✓ value of /*waarde van* *q* **CA** (5)	

11.2



11.2.1	$V = l \times b \times h$ $70\,\text{m}^3 = 3{,}5\,\text{m} \times 10\,\text{m} \times h$ $\therefore h = 2\,\text{m}$	✓ value of h **A** (1)	
11.2.2	$r = \frac{1}{2} \times 3{,}5\,\text{m} = 1{,}75\,\text{m}$	✓ value of r **A** (1)	
11.2.3 #	$\text{SA}_{\text{Rec.Prism}} = 2\times(b\times h)+2\times(l\times h)$ $= 2 \times (3{,}5\,\text{m} \times 2\,\text{m}) + 2 \times (10\,\text{m} \times 2\,\text{m})$ $= 54\,\text{m}^2$ $\text{SA}_{\frac{1}{2}\text{cylinder}} = \frac{1}{2}\times(2\pi r^2 + 2\pi rh)$ $= \frac{1}{2}\left[2\pi(1{,}75)^2 + 2\pi(1{,}75)(10)\right]\text{m}^2$ $= 20{,}56\pi\,\text{m}^2$ **OR/*OF*** $\approx 64{,}60\,\text{m}^2$ $\text{SA}_{\text{T}} = 54\,\text{m}^2 + 64{,}60\,\text{m}^2$ **OR/*OF*** $= 54\,\text{m}^2 + 20{,}56\pi\,\text{m}^2$ $= 118{,}60\,\text{m}^2$ YES, the total surface area is less than 120 m^2 *JA, die totale buite-oppervlakte is minder as* 120 m^2	✓ **SF CA from Q11.2.1** ✓ value of / *waarde van* A_{Prism} **CA** ✓ **SF CA from Q11.2.2** ✓ value of /*waarde van* $\text{A}_{2\frac{1}{2}\text{silinder}}$ **CA** ✓ SA total/*totaal* **CA** ✓ conclusion must be based on total surface area/*gevolgtrekking moet gebaseer word volgens die totale buite-oppervlakte* **CA** (6)	
		[14]	

TOTAL/*TOTAAL*: 150