basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/*SENIOR SERTIFIKAAT*
NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

TECHNICAL MATHEMATICS P1/*TEGNIESE WISKUNDE V1*

NOVEMBER 2020

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

MARKING CODES/*NASIENKODES*	
A	Accuracy/*Akkuraatheid*
AO	Answer only/*Slegs antwoord*
CA	Consistent accuracy/*Volgehoue akkuraatheid*
M	Method/*Metode*
R	Rounding/*Afronding*
NPR	No penalty for rounding/*Geen penalisering vir afronding nie*
NPU	No penalty for omitting unit/*Geen penalisering vir eenhede weggelaat nie*
S	Simplification/*Vereenvoudiging*
F	Correct formula/*Korrekte formule*
SF	Substitution in correct formula/*Vervanging in korrekte formule*

These marking guidelines consist of 26 pages./
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- Consistent accuracy to be applied as indicated on the marking guidelines.
- # Shows questions where a Tolerance Range will be applied are **Q3.1; Q3.3.2; Q4.1.2 &Q5.3.2**

LET WEL:

- *Indien 'n kandidaat 'n vraag **TWEE** keer beantwoord, sien slegs die **EERSTE** poging na.*
- *Volgehoue akkuraatheid sal toegepas word soos op die nasienriglyne aangedui .*
- *# Toon vrae waar Toleransie Wydte (Verdraagsaamheids omvang) toegepas word is **Q3.1; Q3.3.2; Q4.1.2 &Q5.3.2***

QUESTION/*VRAAG* 1

1.1.1(a)	$(12 + 2x)$ **OR/*OF*** $(12 + x + x)$	✓ length/*lengte* **A** (1)
1.1.1(b)	$(3 + 2x)$ **OR/*OF*** $(3 + x + x)$	✓breadth/*breedte* **A** (1)
1.1.2	Area = length × breadth/*lengte* ×*breedte* $= (12 + 2x)(3 + 2x)$ $= 36 + 24x + 6x + 4x^2$ $= 4x^2 + 30x + 36$	✓✓ **SF** **CA** **(2)**
1.1.3	$4x^2 + 30x + 36 = 52$ $4x^2 + 30x - 16 = 0$ **OR/*OF*** $2x^2 + 15x - 8 = 0$ $2(2x-1)(x+8) = 0$ **OR/*OF*** $x = \frac{-(15) \pm \sqrt{(15)^2 - 4(2)(-8)}}{2(2)}$ $x = \frac{1}{2}$ or/of $x \neq -8$ Outside length / *buite lengte* $= 12\,\text{m} + 2\left(\frac{1}{2}\right)\text{m} = 13\,\text{m}$	✓equation/*vergelyking* **CA** ✓factors/formula *faktore/formule* **CA** ✓both *x* values/*beide x-waardes* **CA** ✓length/*lengte* **CA** **NPU** (4)

1.2.1	$\frac{3}{x} = 7x - 5$ $7x^2 - 5x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(7)(-3)}}{2(7)}$ $x = \frac{5 \pm \sqrt{109}}{14}$ $x \approx 1{,}10 \quad or/of \quad x \approx -0{,}39$	✓standard form/ *standaardvorm* **A** ✓ SF **CA** ✓both values of *x/beide x-waardes* **CA** **NPR** (3)
1.2.2	$\therefore x \in \{\text{Real numbers / } \textit{Re ële getalle}\}$ **OR/*OF*** $x \in (-\infty ; \infty)$ **OR/*OF*** $x \in \mathbb{R}$	✓ $x \in$ Real Numbers / *Reële getalle* **A** (1)
1.3	$y - x = 3$ and /*en* $3x^2 + xy - y^2 = -3$ $y = x + 3$ $3x^2 + x(x+3) - (x+3)^2 = -3$ $3x^2 + x^2 + 3x - (x^2 + 6x + 9) + 3 = 0$ $3x^2 - 3x - 6 = 0$ **OR/*OF*** $x^2 - x - 2 = 0$ $3(x-2)(x+1) = 0$ **OR/*OF*** $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $\therefore x = 2$ or /*of* $x = -1$ $y = 2 + 3 = 5$ or /*of* $y = -1 + 3 = 2$ **OR/*OF*** $y - x = 3$ and/*en* $3x^2 + xy - y^2 = -3$ $x = y - 3$ $3(y-3)^2 + y(y-3) - y^2 = -3$ $3y^2 - 18y + 27 + y^2 - 3y - y^2 + 3 = 0$ $3y^2 - 21y + 30 = 0$ **OR/*OF*** $y^2 - 7y + 10 = 0$ $3(y-2)(y-5) = 0$ **OR/*OF*** $y = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$ $\therefore y = 2$ or /*of* $y = 5$ $x = 2 - 3 = -1$ or /*of* $x = 5 - 3 = 2$	✓ subject/*onderwerp* **A** ✓substitution/*vervanging* **CA** ✓ **S** **CA** ✓ factors/*faktore* or/*of* formula **CA** ✓ both *x*-values/ *beide x-waardes* **CA** ✓ both *y*-values/ *beide y-waardes* **CA** **OR/*OF*** ✓subject/*onderwerp* **A** ✓substitution/*vervanging* **CA** ✓**S** **CA** ✓factors/*faktore* or/*of* formula **CA** ✓both *y*-values/*beide y-waardes* **CA** ✓both *x*-values/*beide x-waardes* **CA** (6)

1.4.1	$X_c = \frac{1}{2\pi f C}$ $f = \frac{1}{2\pi X_C C}$ **OR/*OF*** $f = (2\pi X_C C)^{-1}$	✓making *f* the subject/*maak f die onderwerp* **A** (1)
1.4.2	$f = \frac{1}{2\pi X_C C}$ $= \frac{\mathbf{1}}{\mathbf{2}\pi \times 63,66 \times \mathbf{5}0 \times 10^{-6}}$ **OR/OF** $\left(\mathbf{2}\pi \times 63,66 \times \mathbf{5}0 \times 10^{-6}\right)^{-1}$ ≈ 50 hertz **OR/*OF*** $X_c = \frac{1}{2\pi f C}$ $63,66 = \frac{1}{2\pi f \times 50 \times 10^{-6}}$ $f = \frac{1}{2\pi \times 63,66 \times 50 \times 10^{-6}}$ ≈ 50 hertz	✓substitution/*vervanging* **CA** ✓value of/*waarde van* f **CA** **OR/*OF*** ✓substitution/*vervanging* **CA** ✓value of/*waarde van* f **CA** **NPR** **NPU** (2)
1.5.1	$110011_2 + 111101_2 = 1110000_2$ **OR/*OF*** $32 + 16 + 2 + 1 + 32 + 16 + 8 + 1 = 112 = 1110000_2$	✓correct sum/*korrekte som* **A** (1)
1.5.2	<table><tr><td>2^6</td><td>2^5</td><td>2^4</td><td>2^3</td><td>2^2</td><td>2^1</td><td>2^0</td></tr><tr><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table> $= 64 + 32 + 16 = 112$ **OR/*OF*** <table><tr><td>2^5</td><td>2^4</td><td>2^3</td><td>2^2</td><td>2^1</td><td>2^0</td></tr><tr><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr></table> $51 + 61 = 112$	✓**M** **CA** ✓decimal/*desimaal* **CA** **OR/*OF*** ✓**M** **CA** ✓decimal/*desimaal* **CA** **AO: Full marks/ *Volpunte*** (2) **[24]**

QUESTION/*VRAAG* 2

2.1.1	$3x^2+2x+2=0$ $\Delta = b^2-4ac$ $= (2)^2-4\times3\times2$ $= -20$	✓ substitution/*vervanging* **A** ✓value of/*waarde van* Δ **CA** **AO: Full marks/ *Volpunte*** (2)
2.1.2	non-real/ *nie-reël*	✓description/*beskrywing* **CA** **Accept imaginary/ *aanvaar imaginêr*)** (1)
2.2.1	$x^2-2px=3p^2$ $x^2-2px-3p^2=0$ **OR/OF** $-x^2+2px+3p^2=0$	✓standard form/ standaard*vorm* **A** (1)
2.2.2	$\Delta = (-2p)^2-4(1)(-3p^2)$ $\Delta = 4p^2+12p^2$ $=16p^2$ Δ is a perfect square $\therefore$ roots will be rational/ Δ *is volkome vierkant* $\therefore$ *die wortels is rasionaal*	✓subst. in discriminant/ *vervanging in dikriminant* **CA** ✓ **S** **CA** ✓perfect square/ *Volkome vierkant* **CA** (3) **[7]**

QUESTION/*VRAAG* 3

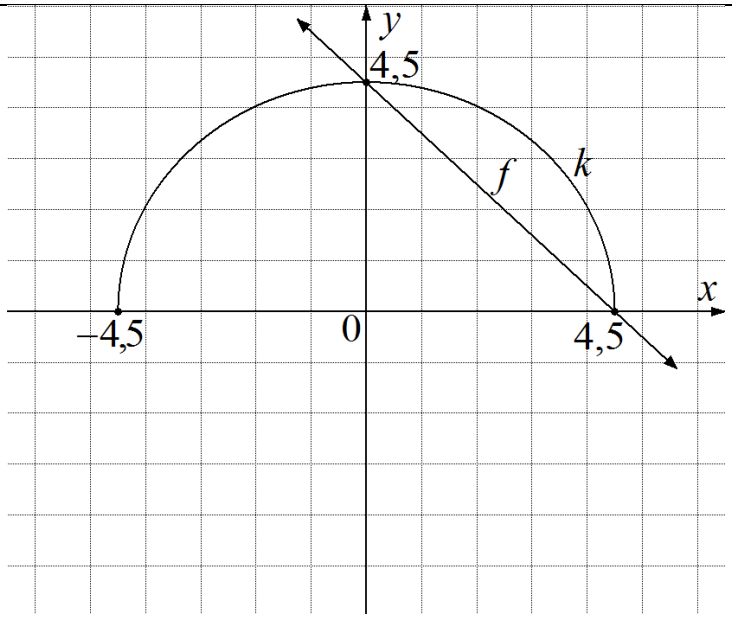
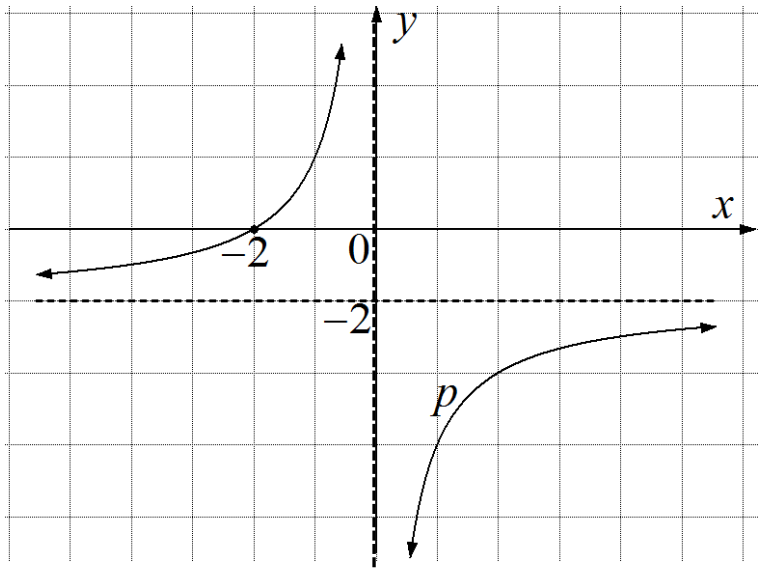
3.1.1 #	$\frac{\log 3 + \log\ 27}{\log 81 - \log 9}$ $=\frac{\log 3+\log 3^3}{\log 3^4-\log 3^2}$ $=\frac{\log 3+3\log 3}{4\log 3\ -2\log 3}$ $=\frac{4\log 3}{2\log 3}$ $=2$ **OR/*OF*** $\frac{\log 3 + \log\ 27}{\log 81 - \log 9}$ $=\frac{\log (3\times 27)}{\log\left(\frac{81}{9}\right)}$ $=\frac{\log 81}{\log 9}$ $=\frac{\log 3^4}{\log 3^2}$ **OR / *OF*** $\frac{\log 9^2}{\log 9}$ **OR / *OF*** $\log_9 81$ $=\frac{4\log 3}{2\log 3}$ **OR / *OF*** $\frac{2\log 9}{\log 9}$ **OR / *OF*** $\log_9 9^2 = 2\log_9 9$ $=2$	✓prime bases/ *Priem grondtalle* **A** ✓log property/*eienskap* **CA** ✓**S** **CA** ✓**S** **CA** **OR/*OF*** ✓log property/*eienskap* **A** ✓ prime bases or log prop/ *Priemgrondtalle of log eienskap* **CA** ✓**S** **CA** ✓**S** **CA** **AO: 1 mark/ *punt*** (4)

3.1.2 #	$\frac{2^n\sqrt{32}+2^n\sqrt{2}}{2^n\sqrt{50}} = \frac{2^n\sqrt{2\times16}+2^n\sqrt{2}}{2^n\sqrt{2\times25}}$ $= \frac{2^n 4\sqrt{2}+2^n\sqrt{2}}{2^n\cdot5\sqrt{2}}$ $= \frac{2^n\sqrt{2}(4+1)}{2^n\cdot5\sqrt{2}}$ $= 1$ **OR/*OF*** $\frac{2^n\sqrt{32}+2^n\sqrt{2}}{2^n\sqrt{50}} = \frac{2^n\sqrt{32}}{2^n\sqrt{50}}+\frac{2^n\sqrt{2}}{2^n\sqrt{50}}$ $= \frac{4\sqrt{2}}{5\sqrt{2}}+\frac{\sqrt{2}}{5\sqrt{2}}$ $= \frac{4}{5}+\frac{1}{5}$ $= 1$ **OR/*OF*** $\frac{2^n\sqrt{32}+2^n\sqrt{2}}{2^n\sqrt{50}} = \frac{2^n\left(2^5\right)^{\frac{1}{2}}+2^n2^{\frac{1}{2}}}{2^n\left(5^2\cdot2\right)^{\frac{1}{2}}}$ $= \frac{2^n2^{\frac{5}{2}}+2^n2^{\frac{1}{2}}}{2^n5\cdot2^{\frac{1}{2}}}$ $= \frac{2^n2^{\frac{1}{2}}\left(2^2+1\right)}{2^n5\cdot2^{\frac{1}{2}}}$ $= 1$ **OR/*OF*** $\frac{2^n\sqrt{32}+2^n\sqrt{2}}{2^n\sqrt{50}} = \frac{2^n\left(\sqrt{32}+\sqrt{2}\right)}{2^n\left(\sqrt{50}\right)}$ $= \frac{4\sqrt{2}+\sqrt{2}}{5\sqrt{2}}$ $= \frac{5\sqrt{2}}{5\sqrt{2}}$ $= 1$	✓simplified surds/*vereenv. wortelvorm* **A** ✓**S** **CA** ✓common factor or like terms */gemene faktor of gelyke terme* **CA** ✓ **S** **CA** **OR/*OF*** ✓ Separating terms/ *skei terme* **A** ✓**S** **CA** ✓ **S** **CA** ✓ **S** **CA** **OR/*OF*** ✓exponent form/*eksponent vorm* **A** ✓**S** **CA** ✓common factor/*gemene faktor* **CA** ✓ **S** **CA** **OR/*OF*** ✓Common factor/*gemene faktor* **A** ✓**S** **CA** ✓common factor/*gemene faktor* **CA** ✓ **S** **CA** **AO: 1 mark/ *Punt*** (4)

3.2	$\log_x 32 + \log_x 4 - \log_x 16 = \log_5 125$ $\log_x \frac{32\times 4}{16} = \log_5 5^3$ $\log_x 8 = 3$ $x^3 = 8 = 2^3$ $\therefore x = 2$ **OR/*OF*** $\log_x 32 + \log_x 4 - \log_x 16 = \log_5 125$ $5\log_x 2 + 2\log_x 2 - 4\log_x 2 = 3\log_5 5$ $3\log_x 2 = 3$ $\log_x 2 = 1$ $\therefore x = 2$ **OR/*OF*** $\log_x 32 + \log_x 4 - \log_x 16 = \log_5 125$ $\log_x \frac{32\times 4}{16} = \log_5 5^3$ $\log_x 8 = 3$ $x^3 = 2^3$ **OR/OF** $\log_x 2^3 = \log_x x^3$ $\therefore x = 2$	✓log property/*eienskap* **A** ✓power form/*magte vorm* **A** ✓**S** **CA** ✓exp form/*eksp. vorm* **CA** ✓value of/*waarde van x* **CA** **OR/*OF*** ✓log property/*eienskap* **A** ✓log identity/*identiteit* **A** ✓**S** **A** ✓**S** **CA** ✓ value of/waarde van *x* **CA** **OR/*OF*** ✓log property/*eienskap* **A** ✓power form/*magte vorm* **A** ✓**S** **CA** ✓exp form/*eksp. vorm* **CA** ✓ value of/*waarde van x* **CA** (5)

3.3.1	$Z_T = 4 + 5i - 4 - 4i$ $= i$	✓total impedance/*totale impedansie* **A** (1)
3.3.2 #	$z_T = i$ $r = 1$ $\tan\theta = \frac{1}{0}$ $\theta = 90°$ **OR/*OF*** $\theta = \frac{1}{2}\pi$ $z_T = 1(\cos 90° + i\sin 90°)$ **OR/*OF*** $z_T = 1\left(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi\right)$	✓value of modulus/ *waarde van modulus* **CA** ✓tan ratio/*verhouding* **CA** ✓correct angle/*korrekte hoek* **CA** ✓z in polar vorm/*polêre vorm* **CA** **AO: 1 mark/*punt*** (4)
3.4	$k = 6 + 4(i - 9) + 2mi$ $k - 2mi = 6 + 4i - 36$ $k - 2mi = -30 + 4i$ $\therefore k = -30$ and/*en* $-2m = 4$ $\therefore k = -30$ and/*en* $m = -2$ **OR/*OF*** $k = 6 + 4(i - 9) + 2mi$ $k - 6 = 4i - 36 + 2mi$ $k = -30 + (2m + 4)i$ $\therefore k = -30$ and/*en* $-2m = 4$ $\therefore k = -30$ and/*en* $m = -2$ **OR/*OF*** $k = 6 + 4(i - 9) + 2mi$ $k - 6 - 2mi = 4i - 36$ $k - 6 - 2mi = -36 + 4i$ $k - 6 = -36$ and/*en* $-2mi = 4i$ $\therefore k = -30$ and/*en* $m = -2$	✓product/*produk* **A** ✓**S** **CA** ✓value of/*waarde van* **k** **CA** ✓value of/*waarde van* ***m*** **CA** **OR/*OF*** ✓product/*produk* **A** ✓**S** **CA** ✓value of/*waarde van k* **CA** ✓value of/*waarde van m* **CA** **OR/*OF*** ✓product/*produk* **A** ✓**S** **CA** ✓value of/*waarde van k* **CA** ✓value of/*waarde van m* **CA** (4) **[22]**

QUESTION/*VRAAG* 4

4.1.1	*radius* / *radius* $= 4,5$ units / *eenhede*	✓length of radius/*lengte van radius* **A** (1)
4.1.2 #		***CA* from/*vanaf* Q/*V4.1.1*** *f:* ✓both intercepts/*beide afsnitte* **CA** ✓negative straight line /*negatiewe reguitlyn* **CA** *k:* ✓*x*-intercepts/*afsnitte* **CA** ✓*y*-intercept/*afsnitte* **CA** ✓semi-circle/ *halfsirkel* **CA** (5)
4.1.3	$x \in [-4,5; 4,5]$ **OR/*OF*** $-4,5 \leq x \leq 4,5$	✓end points/*eindpunte* **CA** ✓correct notation/ *korrekte notasie* **A** (2)
4.2		✓Horizontal asymptote/ *horisontale asimptoot* **A** ✓*x*-intercept/*afsnit* **A** ✓shape/*vorm* (both sections) **A** (3)

4.3.1(a)	T (0 ; 16)	✓coordinates of/ *koördinate van* T **A** (1)
4.3.1(b)	P (– 4 ; 0)	✓– 4 **A** ✓ 0 **A** (2)
4.3.2	$g(x)=a(x-x_1)(x-x_2)$ $g(x)=a(x+4)(x-2)$ $16=a(0+4)(0-2)$ $\therefore a=-2$ $g(x)=-2(x+4)(x-2)$ **OR/*OF*** $-\frac{b}{2a}=-1$ $\therefore g(x)=-2x^2-4x+16$ $-\frac{b}{2(-2)}=-1$ $\therefore\ g'(x)=2ax+b=0$ **OR/OF** $2(-2)(-1)+b=0$ $\therefore\ b=-4$ **OR/*OF*** subst./ *verv.* U (2 ; 0): $0=a(2)^2+b(2)+16$ $4a+2b=-16$ .....(1) subst./ *verv.* S (1 ; 10): $10=a(1)^2+b(1)+16$ $a+b=-6 \Rightarrow 2a+2b=-12$ .....(2) $(1)-(2): 2a=-4$ $\therefore a=-2$ $2(-2)+2b=-12$ $\therefore b=-4$ **OR/*OF***	✓substitution in intercept form/ *vervanging in afsnitvorm* **CA** ✓value of/*waarde van a* **CA** ✓substitution/ *vervanging* **CA** ✓value of/*waarde van b* **CA** **OR/*OF*** ✓substitution/*vervanging* **A** ✓substitution/*vervanging* **A** ✓value of/*waarde van a* **CA** ✓value of/*waarde van b* **OR/*OF***

	$y = a(x+p)^2 + q$ $y = a(x+1)^2 + q$ Subst./verv. (2;0) : $0 = a(2+1)^2 + q$ $0 = 9a + q$.....(1) Subst./verv. (1;10) : $10 = a(1+1)^2 + q$ $10 = 4a + q$.....(2) (1) − (2) $-10 = 5a$ $\therefore a = -2$ $10 = 4a + q$ $10 = 4(-2) + q$ $\therefore q = 18$ $y = -2(x+1)^2 + 18$ $= -2x^2 - 4x - 16$ $\therefore\ b = -4$	✓substitution/*vervanging* **A** ✓substitution/*vervanging* **A** ✓value of/*waarde van a* **CA** ✓value of/*waarde van b* (4)
4.3.3	$g(x) = -2x^2 - 4x + 16$ subst. / *verv.* $x = -1$ $g(-1) = -2(-1)^2 - 4(-1) + 16$ $y = 18$ **OR/OF** (R$(-1;18)$	✓substitution/*vervanging* CA **(Q4.3.2)** ✓ *y*-coordinate of/ *koördinate van* R **CA** **OR/OF** ✓✓ *y*-coordinate of/ *koördinate van* R **CA** (2)
4.3.4	$h(x) = k^x + 8$ $10 = k^1 + 8$ $\therefore k = 2$ $h(x) = 2^x + 8$	✓value of/*waarde van q* **A** ✓substitution/ *vervanging* **A** ✓value of/*waarde van k* **A** (3)
4.3.5	$y > 8$ **OR/OF** $y \in (8;\infty)$	✓range/*waarde-versameling* **A** (1)

4.3.6	subst./ *verv.* $x=-1$ At W: $y=2^{-1}+8=\frac{17}{2}=8,5$ $\text{VW}=\frac{17}{2}-8$ **OR/*OF*** $\text{VW}=8,5-8$ $=0,5$ units / *eenhede* **OR/OF** At W: $y=2^{-1}+8=\frac{17}{2}=8,5$ $\text{VW}=\sqrt{(1-1)^2+(8,5-8)^2}$ $=\sqrt{0,25}$ $=0,5$ units/*eenhede* **OR/*OF*** $h(x)=2^x+8$ eq. of the asympt./ *verg. van asimpt.* $y=8$ $\text{VW}=2^x+8-8=2^x$ $x=-1$ $\therefore \text{VW}=2^{-1}=0,5$ units/eenhede	✓value of/*waarde van y* at/*by* W **A** ✓ **M** **CA** ✓length of/*lengte van* VW **CA** **OR/OF** ✓value of/*waarde van y* at/*by* W **A** ✓ **M** **CA** ✓length of/*lengte van* VW **CA** **OR/OF** ✓value of/*waarde van y* at/*by* W **A** ✓ **M** **CA** ✓length of/*lengte van* VW **CA** **AO: Full marks/ *Volpunte*** (3) **[27]**

QUESTION/*VRAAG* 5

5.1.1	90% of / *van* R 250 000 = R 225 000 **OR/*OF*** 10 % of / *van* R 250 000 = R 25 000 Loan value/*leningswaarde*: R 250 000 – R25 000 = R 225 000	✓Loan value/*waarde van lening* **A** **OR/*OF*** ✓Loan value/*waarde van lening* **A** (1)
5.1.2	$i_{eff.} = \left(1 + \frac{i_{nom.}}{m}\right)^m - 1$ $i_{eff.} = \left(1 + \frac{6,3\%}{12}\right)^{12} - 1$ $\therefore\ i_{eff.} \approx 6,5\%$ **OR/*OF*** $A = P(1+i)^n$ Let/ *Laat* P = R100 $A = 100\left(1 + \frac{6,3\%}{12}\right)^{12}$ = R106, 49 int *erest* / *rente* = *R*106,49 – *R*100 = 6,49 $\therefore i \approx 6,49 \approx 6,5$	✓ **F** **A** ✓ **SF** **A** ✓value of i_{eff} greater than. / *waarde van* i_{eff} groter as 6,3% **CA** **OR/*OF*** ✓ **F** **A** ✓ **SF** **A** ✓ value of i_{eff} greater than. / *waarde van* i_{eff} groter as 6,3% **CA** **AO: Full marks /*Volpunte*** **NPR** (3)

5.2	$A = P(1-i)^n$ $60 = P(1-5,43\%)^4$ $\frac{60}{(1-5,43\%)^4} = P$ $\therefore P \approx 75,01$ $\therefore$ There were 75 unskilled workers during April 2019 *Daar was 75 ongeskoolde werkers gedurende April 2019* **Incorrect formula: one mark for value of *n*/** ***verkeerde formule: een punt vir die waarde van n***	✓**F** **A** ✓ $n = 4$ **A** ✓**SF** **A** ✓ Number of unskilled Workers/ *aantal ongeskoolde werkers* **CA** **Accept/ *aanvaar* 76** **NPR** (4)
5.3.1	Value of the investment at the end of the first 2 years /*waarde van belegging einde van eerste 2 jare :* $A = P(1+i)^n$ $= R\,85\,000\left(1+\frac{5,4\%}{2}\right)^{2\times 2}$ $\approx R\,94558,53$	✓**SF** **A** ✓ $R\,94558,53$ **CA** **NPR** **Incorrect formula: no marks / *verkeerde formule: geen punte*** (2)

5.3.2 #	Value of the investment after change in interest rate for 2 years/ *waarde van belegging na rentekoersverandering vir 2 jaar:* $A = R94558,53\left(1+\frac{6\%}{12}\right)^{2\times12}$ $\approx R106582,57$ Value of the investment after withdrawing/*waarde van belegging na onttrekking:* $P = R106582,57 - R20\,000 = R86582,57$ **YES** it will be more./***JA** dit sal meer wees.* **OR/*OF*** $A = R94558,53\left(1+\frac{6\%}{12}\right)^{4\times12} - 20000\left(1+\frac{6\%}{12}\right)^{2\times12}$ $\approx R\,97592,39$ **YES**, it will be more./***JA** dit sal meer wees.* **OR/*OF*** $A = \left[R94558,53\left(1+\frac{6\%}{12}\right)^{2\times12} - 20000\right]\times\left(1+\frac{6\%}{12}\right)^{2\times12}$ $\approx R\,97592,39$ **YES**, it will be more./***JA** dit sal meer wees.*	**CA from/*vanaf* Q/*V* 5.3.1** ✓✓**SF** **CA** ✓ R106582,57 **CA** ✓M subtracting/ *aftrek* 20000 **A** ✓ difFerence/ *Verskil* **CA** **CA** ✓conclusion/ *gevolgtrekking* **CA** **OR/*OF*** ✓**M** **A** ✓**SF** **CA** ✓ $\left(1+\frac{6\%}{12}\right)^{4\times12}$ **A** ✓ $\left(1+\frac{6\%}{12}\right)^{2\times12}$ **A** ✓ value of/ *waarde van* A_{final} **CA** ✓conclusion/ *gevolgtrekking* **CA** **OR/*OF*** ✓**M** **A** ✓**SF** **CA** ✓✓value of/*waarde van i* and *n* **A** ✓value of/*waarde van* A_{final} **CA** ✓conclusion/ *gevolgtrekking* **CA** **NPR** (6) **[16]**

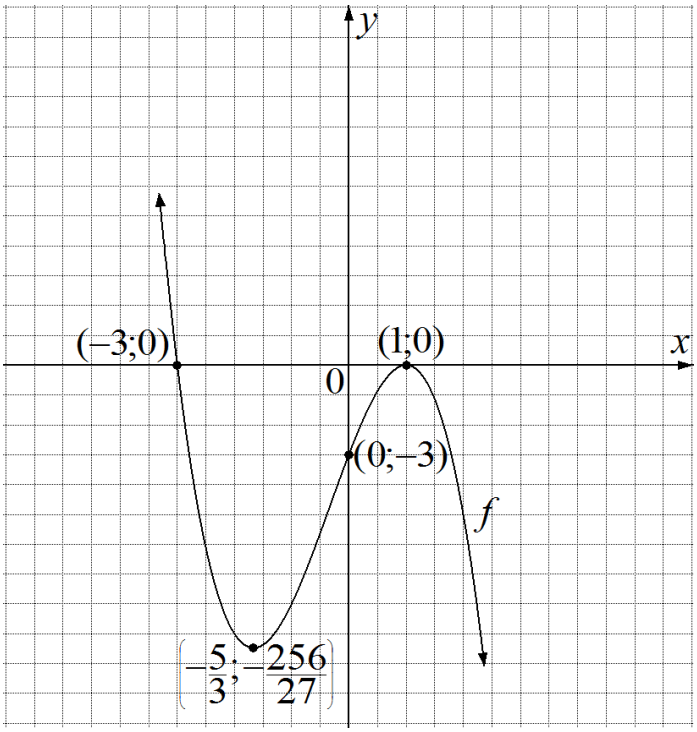
QUESTION/*VRAAG* 6

6.1	$f(x)=\frac{1}{2}x$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\left(\frac{1}{2}(x+h)\right)-\left(\frac{1}{2}x\right)}{h}$ $=\lim_{h\to 0}\frac{\frac{1}{2}x+\frac{1}{2}h-\frac{1}{2}x}{h}$ **OR/*OF*** $=\lim_{h\to 0}\frac{\frac{x}{2}+\frac{h}{2}-\frac{x}{2}}{h}$ $=\lim_{h\to 0}\frac{\frac{1}{2}h}{h}$ **OR/*OF*** $=\lim_{h\to 0}\frac{\frac{1}{2}h}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{1}{2}=\frac{1}{2}$	✓definition/*definisie* **A** ✓**SF** **A** ✓**S** **CA** $\frac{1}{2}$ **CA** **Penalty of 1 mark if incorrect notation used/** ***1 punt penaliseering vir verkeerde notasie*** **AO: 0 marks/ *punte***	(4)
6.2.1	$\frac{dA}{dr}=2\pi r$	✓derivative/*afgeleide* **A**	(1)
6.2.2	$D_x\left[\left(x-\sqrt{x}\right)^2\right]$ $=D_x\left[\left(x-x^{\frac{1}{2}}\right)\left(x-x^{\frac{1}{2}}\right)\right]$ $=D_x\left[x^2-2x^{\frac{3}{2}}+x\right]$ $=2x-3x^{\frac{1}{2}}+1$	✓exponent vorm/ *eksp vorm* **A** ✓**S** **CA** ✓$2x$ **CA** ✓$-3x^{\frac{1}{2}}$ **CA** ✓1 **CA** **No Penalty for incorrect notation used/ *geen penaliseering vir verkeerde notasie*** **AO: 0 marks/ *punte***	(5)

6.3	$g(x) = ax^2 - x \quad \text{sub}\,(-1;-1)$ $-1 = a(-1)^2 - (-1)$ $-1 = a + 1$ $a = -2$ **OR/*OF*** $3x - y + 2 = 0$ $y = 3x + 2$ $\therefore m_{tan/raakl.} = 3$ $g(x) = ax^2 - x$ $g'(x) = 2ax - 1$ $2a(-1) - 1 = 3$ $-2a = 3 + 1$ $a = -2$ **OR/*OF*** $ax^2 - x = 3x + 2$ $ax^2 - 4x - 2 = 0$ $b^2 - 4ac = 0$ (equal roots) tangent touches) $(-4)^2 - 4(a)(-2) = 0$ $16 + 8a = 0$ $a = -2$	✓✓subst/ *verv* **A** ✓ **S** **CA** ✓✓ value of/ waarde van *A* **CA** **OR/*OF*** ✓gradient of tan./*grad van rklyn* **A** ✓derivative/*afgeleide* **A** ✓ $g'(x) = 3$ **CA** ✓substitution/*vervanging* **CA** ✓value of/ *waarde van a* **CA** **OR/*OF*** ✓ equating/ *vergelyk* **A** ✓ std form/ *vorm* **A** ✓ equal roots/ *gelyke wortels* **CA** ✓ **S** **CA** ✓value of/ *waarde van a* **CA** (5) **[15]**

QUESTION/*VRAAG* 7

7.1	$y = f(0) = -(0-1)^2(0+3) = -3$ **OR/*OF*** (0 ; –3)	✓ *y*-intercept / *Afsnit* **A** (1)
7.2	$f(x) = -(x-1)^2(x+3)$ $x = 1$ or/*of* $x = -3$ **OR/*OF*** (1 ; 0) or/*of* (-3 ;0)	✓ $x = 1$ **A** ✓ $x = -3$ **A** (2)
7.3	$f(x) = -x^3 - x^2 + 5x - 3$ $f'(x) = -3x^2 - 2x + 5$ $-3x^2 - 2x + 5 = 0$ $3x^2 + 2x - 5 = 0$ $(3x+5)(x-1) = 0$ **OR/*OF*** $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-5)}}{2(3)}$ $x = -\frac{5}{3}$ or/*of* $x = 1$ $y = -\frac{256}{27} \approx -9{,}5$ or / *of* $y = 0$ **OR/*OF***	✓derivative/*afgeleide* **A** ✓ $f'(x) = 0$ **A** ✓factors/formula *faktore/formule* **CA** ✓both values of/*beide waardes van x* **CA** ✓both values of y/ *beide y-warrdes* **CA** **If derivative is first degree then Max 2 marks/** ***Indien afgeleide eerste order dan Mak. 2 punte*** (5)

7.4		✓cubic shape/*vorm* **A** ✓*y*-intercepts/*afsnitte* **CA from/*vanaf QV7*.1** ✓both/albei *x*-intercepts/*afsnitte* **CA from*vanaf QV7*.2** ✓both turning points/ *draaipunte* **CA from*vanaf QV7*.3** (4) **Using calculator to generate table, maximum 3 marks/ *gebruik van 'n sakrekenaar om 'n tabel te genereer: maksimum 3 punte***
7.5	$-\frac{5}{3}<x<1$ **OR/OF** $x\in\left(-\frac{5}{3};1\right)$	**CA from/*vanaf Q/V7*.4** ✓both end points/*beide eindpunte* **CA** ✓ notation/*notasie* **A** (2) **[14]**

QUESTION/*VRAAG* 8

8.1.1	$D(10) = -0{,}5(10)^2 + 20(10)$ $= 150$ m	✓distance/*afstand* **A** **NPU** (1)
8.1.2	velocity $= D'(t) = -t + 20$ $D'(12) = -(12) + 20$ $= 8$ m/s	✓derivative/*afgeleide* **A** ✓substitution in derivative/ *Vervanging*in afgeleide **CA** ✓velocity/*snelheid* **CA** **NPU** (3)
8.2.1(a)	TSA/*TBO*. $= (4x)(3x) + (5x)(y) + (4x)(y) + (3x)(y)$ $= 12x^2 + 5xy + 4xy + 3xy$ $3600 = 12x^2 + 12xy$ $300 = x^2 + xy$ $xy = 300 - x^2$ $\therefore y = \frac{300 - x^2}{x}$ **OR/*OF*** TSA/*TBO* $= (3x + 4x + 5x)y + 2\left(\frac{1}{2}\cdot 3x \cdot 4x\right)$ $12xy + 12x^2 = 3600$ $300 = x^2 + xy$ **OR/OF** $xy = 300 - x^2$ $\therefore y = \frac{300 - x^2}{x}$	✓area/*oppervlakte* **A** ✓equat. area to/*stel oppervl gelyk* 3 600 **A** ✓**S** **CA** **OR/*OF*** ✓ area/*oppervlakte* **A** ✓equat. area to 3 600/*stel oppervl gelyk* **A** ✓**S** **CA** (3)
8.2.1(b)	$V = \frac{1}{2}(3x)(4x)\left(\frac{300 - x^2}{x}\right)$ $= 6x(300 - x^2)$ $= 1800x - 6x^3$	✓**SF** **CA** ✓**S** **CA** (2)
8.2.2	$V = 1800x - 6x^3$ $\frac{dV}{dx} = 1800 - 18x^2$ $1800 - 18x^2 = 0$ **OR/*OF*** $18(100 - x^2) = 0$ $x^2 = 100$ $\therefore x = 10$	✓derivative/*afgeleide* **CA** ✓equating derivative to 0/*gelykstel van afgeleide aan* 0 **A** ✓ value of/*waarde van x* **CA** (3) **[12]**

QUESTION/*VRAAG* 9

Penalize for constant C in either Q 9.1.1 or Q 9.1.2 / ***Penaliseer vir konstante C in of V9.1.1 of V9.1.2***		
9.1.1	$\int 2^x\,dx$ $= \frac{2^x}{\ln 2} + C$	✓ $\frac{2^x}{\ln 2}$ **A** ✓ C **A** (2)
9.1.2	$\int\left(\sqrt{x} + \frac{7}{x} + 4x^{-5}\right)dx$ $= \int\left(x^{\frac{1}{2}} + \frac{7}{x} + 4x^{-5}\right)dx$ $= \frac{2}{3}x^{\frac{3}{2}} + 7\ln x - x^{-4} + C$ **OR/*OF*** $= \frac{2}{3}x^{\frac{3}{2}} + 7\ln x - \frac{1}{x^4} + C$ **OR/*OF*** $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 7\ln x - \frac{1}{x^4} + C$	✓power vorm/*magte vorm* **A** ✓ $\frac{2}{3}x^{\frac{3}{2}}$ OR/*OF* $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ **CA** ✓ $7\ln x$ **A** ✓ $-x^{-4}$ **OR / *OF*** $-\frac{1}{x^4}$ ..... **A** (4)

9.2	Area above the x-axis/ *oppervlakte bo die x-as*: $= \int_{-3}^{2}\left(-x^2 - x + 6\right)dx$ $= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x\right]_{-3}^{2}$ $= \left(-\frac{(2)^3}{3} - \frac{(2)^2}{2} + 6(2)\right) - \left(-\frac{(-3)^3}{3} - \frac{(-3)^2}{2} + 6(-3)\right)$ $= \frac{125}{6}$ square units/*vk.eenhede* Unshaded area / *ongearseerde oppvlk* $= \frac{125}{6} - \frac{34}{3}$ $= \frac{19}{2}$ square units / *vk.eenhede* $\therefore$ the unshaded area **is LESS** than the shaded area/*Die ongearseerde oppvlk **is MINDER** as die gearseerde oppervlakte.* **OR/*OF***	✓area notation using integrals/*oppervl notasie deur integrale* **A** ✓integration/*integrasie* **A** ✓✓ subst./*verv.* **CA** ✓**S** **CA** ✓**M** unshaded area/ *ongearseerde oppvlk* **CA** ✓conclusion/*gevolgtrkng* **CA** **OR/*OF***

	Unshaded area above the x - axis/*ongearseerde oppvlk bo die x - as*: $=\left[\int_{-3}^{-1}\left(-x^2-x+6\right)dx\right]+\left[\int_{1}^{2}\left(-x^2-x+6\right)dx\right]$	✓area notation using Integrals/*oppervl notasie deur integrale* **A**
	Unshaded area/*ongearseerde oppvlk* 1: $=\left[-\frac{x^3}{3}-\frac{x^2}{2}+6x\right]_{-3}^{-1}$	✓integration/*integrasie* **A**
	$=\left[\left(-\frac{(-1)^3}{3}+-\frac{(-1)^2}{2}+6(-1)\right)-\left(-\frac{(-3)^3}{3}-\frac{(-3)^2}{2}+6(-3)\right)\right]$	✓ subst./*verv.* **CA**
	$=\frac{22}{3}$ square units/*vk.eenheede*	\
	Unshaded area/*ongearseerde oppvlk* 2: $=\left[-\frac{x^3}{3}-\frac{x^2}{2}+6x\right]_{1}^{2}$	
	$=\left[\left(-\frac{(2)^3}{3}-\frac{(2)^2}{2}+6(2)\right)-\left(-\frac{(1)^3}{3}-\frac{(1)^2}{2}+6(1)\right)\right]$	✓ subst./*verv* **CA**
	$=\frac{13}{6}$ square units/*vk.eenhede*	✓ **S** **CA**
	$\therefore$ Totalunshaded area $=\frac{22}{3}+\frac{13}{6}=\frac{19}{2}$ square units	✓**M** unshaded area/ *ongearseerde opperv* **CA**
	$\therefore$ The unshaded area **is LESS** than the shaded area/ *Die ongearseerde oppervlakte is **MINDER** as die gearseerde oppervlakte.*	✓conclusion/*gevolgtrkng* **CA** **AO (conclusion *gevolgtrekking*): 1 mark/ *punt*** (7) **[13]**

TOTAL/*TOTAAL*: 150

ADDENDUM

TECHNICAL MATHEMATICS/ *TEGNIESE WISKUNDE*
Paper 1/ *Vraestel 1* November 2020

FINAL MARKING GUIDELINES (ADDITIONAL NOTES)
FINALE NASIEN RIGLYNE (ADDISIONELE NOTAS)

ITEM	DESCRIPTION/ *VERKLARING*
1.1.2	Factors must have a variable *x* and product should lead to a quadratic equation. *Faktore moet 'n onbekende x en die produk moet lei na 'n kwadratiese vergelyking*
1.1.3	If $4x^2+30x-16=0$ is used and leading to negative *x*-values with not valid conclusion, maximum 2 marks *Indien* $4x^2+30x-16=0$ *gebruik word en lei na negatiewe x-waardes met nie geldige gevolgtrekking, maksimum 2 punte*
1.2.1	Linear equation, no marks */ liniêre vergelyking: geen punt*
1.3	If simplification leads to linear equation,maximum 3 marks *Indien vereenvoudiging lei na 'n liniêre vergelyking; maksimum 3 punte*
1.5.1	If base 2 is omitted,no penalty / *indien grondtal 2 uitgelaat is; geen penalisering*
2.2.1	Order of terms not necessary/ *orde van terme nie belangrik*
2.2.2.	• If p is omitted,accept $\Delta=16$,maximum 2 marks/ *indien p uitegelaat is aanvaar* $\Delta=16$ *maksimum 2 punte* • If Δ is irrational based on **CA** from **Q2.2.1**, maximum 3 marks/ *Indien* Δ *irrasionaal is gebaseer op **CA** vanaf **Q2.2.1**, maksimum 3 punte*
3.3.2 #	• $\tan\theta=\frac{1}{0}$ can be implied/ *kan geïmpliseer word* • If 1 is omitted, no penalty/ *indien 1 uitgelaat is, geen penaliseering* • Accept/ *aanvaar* $Z_T=1cis\,90^o$

<table>
<tr><th>ITEM</th><th colspan="2">DESCRIPTION/ VERKLARING</th></tr>
<tr><td rowspan="2">5.2</td><td>• If:
Year 1: $60 \div (1-5,3\%) = 63,45$
Year 2: $63,45 \div (1-5,3\%) = 67,09$
Year 3: $67,09 \div (1-5,3\%) = 70,94$
Year 4: $70,94 \div (1-5,3\%) = 75,01$
$\therefore$ 75 workers/werkers</td><td>✓ F
✓ SF
✓ $n = 4$
✓ Number of skilled workers/ aantaal geskoolde werkers
Maximum 4 marks/ maksimum 4 punte</td></tr>
<tr><td colspan="2">• Simple interest used , maximum 3 marks/Enkelvoudige rente; maksimum 3 punte
• Depreciation Compound used,maximum 4 marks/ Waardevermindering gebruik dan maksimum 4 punte</td></tr>
<tr><td>7.3</td><td colspan="2">If derivative is first degree,maximum 2 marks/ Indien afgeleide 'n eerstegraadsvergelyking is maksimum 2 punte</td></tr>
<tr><td>7.4</td><td colspan="2">• If point by point plotted and Turning Point not shown maximum 3 marks/ indien punt-vir-punt geplot en draaipunt nie getoon maksimum 3 punte
• If 2 Turning points are correctly plotted, award a mark./ As twee draaipunte korrek geplot is, gee 'n punt</td></tr>
</table>