Basic Education

KwaZulu-Natal Department of Basic Education
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MATHEMATICS P2

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MARKING MEMORANDUM

NATIONAL
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GRADE 11

MARKS : 100

This memorandum consists of 8 pages.

Symbol	Explanation
CA	Consistent accuracy
A	Accuracy
S	Statement
R	Reason
S/R	Statement with reason

QUESTION1

1.1.1 Q(1;3)	1A for answer (1)
1.1.2 S(−4;−2)	1A for answer (1)
1.1.3 $y=3$	1A for answer (1)
1.1.4 $x=1$	1A for answer (1)
1.2.1 $\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $=\left(\frac{0+4}{2};\frac{4+(-2)}{2}\right)$ ✓ $=(2;1)$ ✓	 1A for substitution 1CA for answer (2)
1.2.2 $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(4-0)^2+(-2-4)^2}$ ✓ $=\sqrt{52}$ $=7{,}21$ ✓	 1A for substitution 1CA for answer (2)
1.2.3 $m_{BK}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{1-(-3)}{2-(-4)}$ ✓ $=\frac{4}{6}$ $=\frac{2}{3}$ ✓ $m_{AC}=\frac{-2-4}{4-0}$ ✓ $=\frac{-6}{4}$ $=-\frac{3}{2}$ ✓ $m_{BK}\times m_{AC}=\frac{2}{3}\times-\frac{3}{2}=-1$ ✓ $\therefore BK\perp AC$ or $B\hat{K}C=90°$	 1CA for substitution 1CA for value of m_{BK} 1CA for substitution 1CA for value of m_{AC} 1A for showing that product of two gradients equals – 1, and concluding (5)

1.2.4 $y = mx + c$ $= \frac{2}{3}x + c$ ✓ Substitute $(-4;-3)$: $-3 = \frac{2}{3}(-4) + c$ ✓ $c = -3 + \frac{8}{3} = -\frac{1}{3}$ $y = \frac{2}{3}x - \frac{1}{3}$ ✓ OR $y - y_1 = m(x - x_1)$ $= \frac{2}{3}(x - x_1)$ ✓ Substitute $(-4;-3)$: $y - (-3) = \frac{2}{3}[x - (-4)]$ ✓ $y + 3 = \frac{2}{3}x + \frac{8}{3}$ $y = \frac{2}{3}x - \frac{1}{3}$ ✓	1CA for substitution of gradient of BK 1CA for substitution of coordinates of B (or K) 1CA for answer (3) OR 1CA for substitution of gradient of BK 1CA for substitution of coordinates of B (or K) 1CA for answer (3)
1.2.5 Length of BK $= \sqrt{[2-(-4)]^2 + [1-(-3)]^2}$ ✓ $= \sqrt{6^2 + 4^2}$ $= \sqrt{52}$ $= 7{,}21$ ✓ Area of ΔABC $= \frac{1}{2} \times \text{base} \times \text{height}$ ✓ $= \frac{1}{2} \times AC \times BK$ $= \frac{1}{2} \times 7{,}21 \times 7{,}21$ ✓ $= 25{,}99$ ✓	1CA for substitution 1CA for answer to length of BK 1A for formula 1CA for substitution 1CA for answer (also accept: 26) (5)

1.2.6 $m_{AT} = \tan 153{,}4°$ $= -\tan 26{,}6°$ $= -0{,}50$ ✓ Substitute (0; 4) and find equation of AT: $y = mx + c$ $4 = -0{,}50(0) + c$ ✓ $c = 4$ $y = -0{,}50x + 4$ ✓ S lies on the line: $1 = -0{,}50(p) + 4$ ✓ $p = 6$ ✓	1A for gradient of AT 1CA for substitution of (0; 4) and gradient 1CA for equation 1CA for substitution of (p; 1) into equation 1CA for answer (5) **[26]**

QUESTION 2

2.1.1 $m_{RQ} = m_{PQ} = -1$ ✓	1A for equating gradients (1)
2.1.2 $y - y_1 = m(x - x_1)$ $y - 2 = -1(x - 3)$ ✓ $y = -x + 5$ ✓ **OR** $y = mx + c$ $2 = (-1)(3) + c$ ✓ $c = 5$ $y = -x + 5$ ✓	1A for substitution 1CA for equation **OR** 1CA for substitution 1CA for answer (2)
2.1.3 $-x + 5 = \frac{1}{4}x$ ✓ $4x + 20 = x$ ✓ $-5x = -20$ $x = 4$ ✓ $y = 1$ ✓ Q(4; 1)	1CA for equating the equations 1 CA for simplification 1CA for value of x 1CA for value of y (4)

2.1.4	$\tan X\hat{P}Q = \frac{1}{4}$ ✓ $X\hat{P}Q = 14{,}04°$ ✓ $\tan X\hat{P}S = -1$ $X\hat{P}S = 135°$ ✓ $S\hat{P}Q = 135° - 14{,}04°$ ✓ $S\hat{P}Q = 120{,}96°$ ✓	1A for substitution in correct equation 1A for correct angle 1A for correct angle 1CA for subtraction 1CA for answer (5)
2.2.1	$AB = \sqrt{(6-0)^2 + (7+1)^2}$ ✓ $= 10$ ✓	1A for substitution 1A for answer (2)
2.2.2	$AB = 2\,BC$ $AB^2 = (2\,BC)^2$ ✓ $100 = 4[(0-4)^2 + (-1-p)^2]$ ✓ ✓ $100 = 68 + 8p + 4p^2$ $4p^2 + 8p - 32 = 0$ ✓ $4(p^2 + 2p - 8) = 0$ $(p+4)(p-2) = 0$ $p = -4$ or $p = 2$ $p = -4$ ✓	1M for squaring 1A for substitution of AB^2 1A for substitution of $(2BC)^2$ 1CA for simplification 1CA for value of p
OR	$BC = \frac{1}{2}AB$ $= 5$ ✓ $5 = \sqrt{(0-4)^2 + (-1-p)^2}$ ✓ $25 = (0-4)^2 + (-1-p)^2$ ✓ $25 = 4^2 + 1^2 + 2p + p^2$ $p^2 + 2p - 8 = 0$ ✓ $(p+4)(p-2) = 0$ $p = 2$ or -4 $p = -4$ ✓	**OR** 1CA length of BC 1CA for substitution 1CA squaring 1CA simplification 1CA value of p (5) **[19]**

QUESTION 3

3.1	is perpendicular to the chord ✓	1S for correct conclusion (1)
3.2.1	$AP = 8$ ✓ (line from centre of circle to midpoint of chord) ✓	1S ; 1R (2)
3.2.2	$AO^2 = OP^2 + AP^2$ (Pythagoras) ✓ $(x+2)^2 = x^2 + 8^2$ ✓ ✓ $x^2 + 4x + 4 = x^2 + 64$ ✓ $4x = 60$ $x = 15$ radius $= 15 + 2 = 17$ ✓	1S/R for Pythagoras 2A for correct substitution 1CA for simplification 1CA for answer (5) **[8]**

QUESTION 4

4.1	Construction: Draw AO and extend to D ✓ Proof: Let $B\hat{A}O = x$ $OB = OA$ (radii) $\therefore \hat{B} = x$ (angles opposite equal sides) ✓ $\therefore \hat{O}_2 = 2x$ (exterior angle of Δ) ✓ Similarly let $C\hat{A}O = y$ $OA = OC$ (radii) $\therefore \hat{C} = y$ (angles opposite equal sides) $\therefore \hat{O}_1 = 2y$ ✓ (exterior angle of Δ) But $\hat{O}_1 + \hat{O}_2 = 2x + 2y$ ✓ $\therefore B\hat{O}C = 2(x+y)$ ✓ $= 2\,B\hat{A}C$ $= 2\hat{A}$	1M for correct construction 1S/R 1S/R 1 for $\hat{O}_1 = 2y$ 1 for $\hat{O}_1 + \hat{O}_2 = 2x + 2y$ 1 for $B\hat{O}C = 2(x+y)$ (6)

4.2.1(a)	$\hat{C}_6 = \hat{F} = x$ ✓ (FA = CA; angles opposite equal sides) ✓ $\hat{A}_1 = 2x$ ✓ (exterior angle of Δ) ✓	1S ; 1R 1S ; 1R (4)
4.2.1(b)	$D\hat{O}C = 4x$ ✓ ($\angle$ at centre = 2 x $\angle$ at circumference) ✓	1S ; 1R (2)
4.2.2	DA = CA (both = FA) $\therefore A\hat{D}C = A\hat{C}D$ ✓ (angles opposite equal sides) $= \frac{180° - \hat{A}_1}{2}$ ✓ (sum of $\angle$s of ΔADC) ✓ $= \frac{180° - 2x}{2}$ $= 90° - x$ $B\hat{C}D = 90° - x + x$ ✓ $= 90°$ $\hat{A}_3 = 90°$ ✓ (exterior $\angle$ of cyclic quad) ✓	 1S 1S; 1R 1S for calculating $B\hat{C}D$ 1S ; 1R (6) **[18]**

QUESTION 5

5.1.1	$\hat{K}_1 = 90°$ (angle in a semicircle) ✓	1S/R (1)
5.1.2	$\hat{G} = 180° - (\hat{K}_1 + \hat{H}_1)$ (sum of angles of Δ) $= 180° - (90° + 26°)$ ✓ $= 74°$ $\hat{J} = 180° - \hat{G}$ (opposite $\angle$'s of cyclic quad) ✓ $= 180° - 74°$ $= 106°$ ✓	 1S 1S/R 1 answer (3)
5.2.1	$\hat{G} = \hat{E}_1 = 28°$ ✓ (angles in the same segment) ✓	1S ; 1R (2)
5.2.2	$\hat{C}_1 = \hat{E}_2 = 64°$ ✓ (exterior $\angle$ of cyclic quad) ✓ $\hat{B} = 180° - (\hat{G} + \hat{C}_1)$ (sum of $\angle$s of Δ) ✓ $= 180° - (28° + 64°)$ $= 88°$ ✓	1S ; 1R 1S/R 1 answer (4) **[10]**

QUESTION 6

6.1	$\hat{Q}_2 = \hat{R}_2 = x$ ✓ (tan-chord-theorem) ✓ $\hat{P}_2 = \hat{R}_2 = x$ ✓ (alt. $\angle$s; AR ∥ PT) ✓ $\therefore \hat{P}_2 = \hat{Q}_2$ (both = x) PTRQ is a cyclic quadrilateral (converse: angles in the same segment) ✓	1S ; 1R 1S ; 1R 1R (5)
6.2.1	$\hat{Q}_2 = \hat{R}_3 = x$ (angles opposite equal sides) ✓ $P\hat{R}S = 180° - (\hat{R}_2 + \hat{R}_3)$ (QRS is a straight line) $= 180° - 2x$ ✓ $\hat{S} = 180° - (180° - 2x + x)$ (sum of $\angle$s of ΔPRS) $= x$ ✓	1S/R 1S 1S (3)
6.2.1	$\therefore \hat{P}_2 = \hat{S}$ ✓ (both = x) $\therefore$ PR = RS (sides opposite = $\angle$s) ✓ But PR = PQ ✓ (2 tangents from same point) ✓ $\therefore$PQ = RS ✓	1S 1S/R 1S; 1R 1S (5)
6.2.3	$\hat{Q}_1 = \hat{R}_3 = x$ (tan chord theorem) ✓ $\therefore P\hat{Q}R = 2x$ ✓ ($Q_2 = x$; already proved) $P\hat{Q}R = \hat{T}_3$ (exterior $\angle$ of cyclic quad) ✓ $\therefore \hat{T}_3 = 2x$ $\hat{A}_2 = \hat{Q}_2 + \hat{R}_3$ (exterior angle of Δ) ✓ $= x + x$ $= 2x$ $\hat{T}_3 = \hat{A}_2$ (both = $2x$) ✓ PTS is a tangent to circle TAR (converse: tan chord theorem) ✓	1S/R 1S 1S/R 1S/R 1S 1R
OR	$\hat{Q}_1 = \hat{R}_3 = x$ (tan chord theorem) ✓ $\hat{Q}_1 = \hat{R}_1 = x$ (angles in the same segment) ✓ $T\hat{R}A = \hat{R}_1 + \hat{R}_2 = 2x$ ✓ But $\hat{T}_1 = \hat{R}_2 + \hat{R}_3 = 2x$ (angles in the same segment) ✓ $\therefore \hat{T}_1 = T\hat{R}A$ (both = $2x$) ✓ $\therefore$PTS is a tangent to circle TAR (converse: tan chord theorem) ✓	**OR** 1S/R 1 S/R 1S 1S/R 1S 1R (6) **[19]** **TOTAL: 100**